The Trouble With Math is English

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Objectives

1) Examine how mathematics language and symbolism impact students' understanding.

2) Examine the merger of mathematics language and symbolism with difficulties in mathematics instruction.

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Connecting Mathematics and Language

In one word, state what comes to mind for most people when you say "mathematics."

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Connecting Mathematics and Language

- Imagine you are a sophomore taking the TAKS and you come upon problem number 14, below.
- Solve it and discuss with others at your table.

14) Find the *ugloft* of a *bipkad* if the *rexnuza* is 20.

Find the *ugloft* of a *bipkad* if the *rexnuza* is 20.

- You look on the information sheet of the TAKS test and see the following:

ugloft	=	area
bipkad	=	circle
rexnuza	=	diameter

- Now solve the problem. What made the difference?

Connecting Mathematics and Language

- 1) Solve:

$$\sum_{n=1}^{4} 2n - 1$$

Connecting Mathematics and Language

- 2) Solve:

$$X = 1 + 3 + 5 + 7$$

"I kill cats and eat them."

Reaction/Interpretation?

"I kill cats and eat them."

Mathematics and Language

It is not an ELL problem. (ELL students)

It is an ALL problem. (ALL students)

And ALL students are MLLs.

Mathematics and Language

There are many factors that contribute to language being problematic in mathematics:

Total isolation of mathematics and language arts

Learning the vocabulary is part of the learning

Difficulty of the English language itself

Mathematics and Language

(Factors, continued)

- Mathematical terms are rarely used outside of the mathematics classroom
- Abstract nature of many mathematical terms
- **Ambiguity**

Difficulty of the English Language

- ✓Extra
- ✓Ordinary
- ✓Extraordinary

Difficulty of the English Language

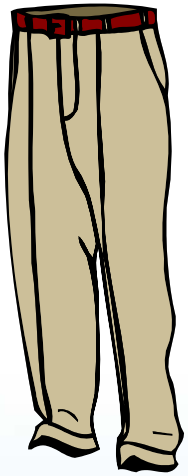
- ✓Logarithm
- ✓Algorithm
- ✓Paradigm

Difficulty of the English Language

- Hot water heater
- Water heater
- 30 – second exercise
- 32nd exercise

Difficulty of the English Language

(Whose idea was this?)



Ambiguity

Reduce:

6

8

Answer: **6**
8

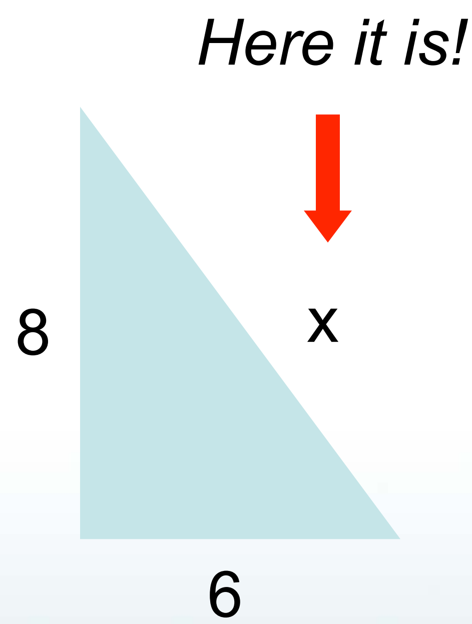
Ambiguity

Expand: **(x + 3) (x – 4)**

Answer: **(x + 3) (x - 4)**

Ambiguity

Find x.



Ambiguity

1) Which is larger?

37 **5**

Ambiguity

2) Which number is larger?

37 **5**

Ambiguity

3) Which numeral is larger?

37 **5**

Ambiguity

1. Which is larger?
2. Which number is larger?
3. Which numeral is larger?

An example of
- how language details are important and
- how ambiguity exists *within* mathematics

Mas Ambiguity I

Additional Confusion

"Number" can still be about mathematics, but not refer to a quantity (e.g., number as a name, code, or location).

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Mas Ambiguity II

Confusion — Pronunciation

A) What are some ways to say 2632?

Two thousand six hundred thirty two
Twenty six hundred thirty two
Twenty six thirty two

B) How can these two addresses be confused?

3218 Main Street & 30218 Main Street

Mas Ambiguity III

Confusion — Pronunciation

C) 2560 and $25.60

How are these often pronounced and what is problematic about it?

D) 25.6

How is this often pronounced and what is problematic about it?

Mas Ambiguity IV

A big idea in mathematics can have several terms associated with it that must also be learned. There may be added complexity if there is ambiguity or confusion with the true meaning of those additional terms.

- Example: **Number**
- Related terms: numeral amount place value digit quantity

Multiple Meanings

Part of the difficulties occur because of polysemous words whose multiple meanings can cause confusion.

Polysemous words can have multiple meanings within mathematics or have one meaning in mathematics and another in standard English.

Even More Ambiguity!

There is also the problem of mathematical terms having *other meanings outside of mathematics.* Students (and teachers) need to decipher the meanings of words in mathematics from the meaning in standard English.

Ex. Degree

Translation

Even More Ambiguity I

Number

- The opening number was the highlight of the show.
- His last girlfriend really did a number on him.
- My finger is number than it was five minutes ago.

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Even More Ambiguity II

If that wasn't enough, we make mathematics language more difficult because of. . .

1) "fluid" mathematical terms — invent "new" names or change meanings in mathematics terminology.

Ex: $-(-x) = x$ The Op-Op property

Commutative property = "Order property"

"FOIL" Method

New definition of "average"

Trapezoid

Even More Ambiguity II

2) elementary or middle school definitions with new meanings in higher level mathematics.

Ex. Exponent ($7^{1/2}$)

3) more than one definition or meaning for the same math terms.

Ex. ounce (weight or volume?)

inverse (function, operation, multiplicative and additive, matrix, variation, proportion, trig. function, etc.)

Traditional Instruction and Language

"Traditional" instruction is characterized by

1) teacher lecture,
2) passive students,
3) repeated drill and practice,
4) memorization, and
5) an emphasis on answers rather than explanations.

Traditional Instruction and Language

Many educators experienced "traditional instruction" and studies indicate that teachers tend to teach in the same way as they were taught.

What would this tendency to teach as we were taught include?

Traditional Instruction and Language

- This tendency to teach as we were taught includes the associated language and symbols. In addition, we not only tend to teach *HOW* we were taught, but also *WHAT* we were taught.

- Traditional instruction pays little attention to mathematics language and symbolism and their impact on learning. In many ways, traditional instruction not only perpetuates, but also adds to the language-based problems in mathematics instruction.

Traditional Instruction and Language

Traditional Instruction and Language

Language-based problems in mathematics instruction include the following:

I) Use of mathematical and non-mathematical meanings at the same time

II) Use of spatial words when describing arithmetical operations

III) Confused logic and mismatched symbolism

Traditional Instruction and Language

(Language-based problems, continued):

IV) The use of “shortcuts”

V) Teachers’ tendency to use “careless” vocabulary

VI) Dominant use of “naked” numbers

Careless Vocabulary

Teachers have a tendency to use the same terms that were used when they learned mathematics. Some of it is “careless” vocabulary. This often involves using terms that have other meanings in standard English.

Careless Vocabulary

Consider this expression: 6 x 7

- How is that usually pronounced?
- Is that problematic? If so, why?

Careless Vocabulary

- 6 x 7 is pronounced as 6 *times* 7.
- In most contexts this represents 6 groups of 7, which is actually 7 + 7 + 7 + 7 + 7 + 7
- So shouldn't this really be described as 7 *six* times, as opposed to 6 *times* 7?

Careless Vocabulary

- How about this as 6 *sevens*?
- Would it make more sense for students to see 6y as six "y's" instead of 6 times y?

(Why do we even call it the *times* table?)

Careless Vocabulary

- Makes sense for 6 x 7 to mean 6 sets of 7.
- Students can make the algebraic connection to an expression: $3(y + 5)$.
- $3(y + 5)$ is 3 sets of $(y + 5)$, or
  $(y + 5) + (y + 5) + (y + 5)$
- Eliminates the common mistake that
  $3(y + 5) = 3y + 5$

Careless Vocabulary

Discuss with others at your table what is problematic with the following (for both ELL and ALL students!)

a) Give your little brother the "***bigger half***" of the cookie

b) **"*Carry*"** the one

c) ***"Borrow"*** a ten

Careless Vocabulary

Discuss with others at your table what is problematic with the following (for both ELL and ALL students!)

d) **"*Cancel*"** when simplifying a fraction or rational expression

e) **"*Reduce*"** a fraction

f) Two ***"goes into"*** eight

Careless Vocabulary

Solve mentally:

$$\frac{10}{\frac{1}{2}} = ?$$

How much sense does this question make: *"How many times does ½ go into 10?"*

Use of "Naked" Numbers

About this problem……

Solve mentally:

$$\frac{10}{\frac{1}{2}} =$$

Did we leave the answer as JUST 20, or did we clarify that the answer was 20 *halves*?

Use of "Naked" Numbers

Given: 25 x 28 and 24 x 28

How much more is the first product?

What will most students do to answer the question and why is that?

Use of "Naked" Numbers

Connecting back to 6 x 7 … taking this beyond "how to" multiply—

Students are not usually taught to see numerals also as words, which would enable them to interpret the above as 6 *sevens*.

25 ***cows*** is 1 more ***cow*** than 24 ***cows****.*

25 ***twenty-eights*** is 1 more ***twenty-eight*** than 24 ***twenty-eights*****.**

Language, Symbols, and Instruction

"The difference between the right word and the almost-right word is like the difference between lightning and lightning bug."

Mark Twain

Part of teachers' content knowledge is an awareness of how their use of mathematical terms, symbolism, and non-mathematical vocabulary impact student learning and understanding.

Classroom Application

What can we do to teach MATHEMATICS (not arithmetic and efficiency) to ALL students?

- To the extent possible, use concrete examples or manipulatives
- Emphasize both symbolism and academic language.
- Organize thinking with graphic organizers
- Use ambiguity to your advantage, not a disadvantage.

Classroom Application

What can we do to teach MATHEMATICS (not arithmetic and efficiency) to ALL students?

- Simplify, yet deepen.
- Use the deep knowledge in one topic to make connections and to leverage the learning of related topics.
- Teach mathematics as relationships.

Emphasize Both Symbolism and Language

A map indicates the following scale:

1 inch = 20 miles

Would that confuse you?

Consider: $\frac{\text{1 inch}}{\text{20 miles}}$ (on map) / (real life)

Emphasize Both Symbolism and Language

Students' Interpretation of Symbols

1) Do students see $\frac{5}{4}$ as $5 \times \frac{1}{4}$?
Do students see $\frac{5}{4}$ as 5 *one-fourths*?
Why or why not?

2) Do students see $1\frac{1}{2}$ as $1 + \frac{1}{2}$?
Do students see that $9 = 1\frac{1}{2} \times 6$ means that 9 is 1 and $\frac{1}{2}$ *sixes* (1 six and half of another six)?

Ambiguity as an Asset

How can we use ambiguity as an asset in our classroom instruction rather than it being a liability?

Examples:

1) One pig grew from 5 pounds to 10 pounds. Another grew from 100 to 108 pounds. Which pig grew more?

2) Are a square and a rectangle similar?