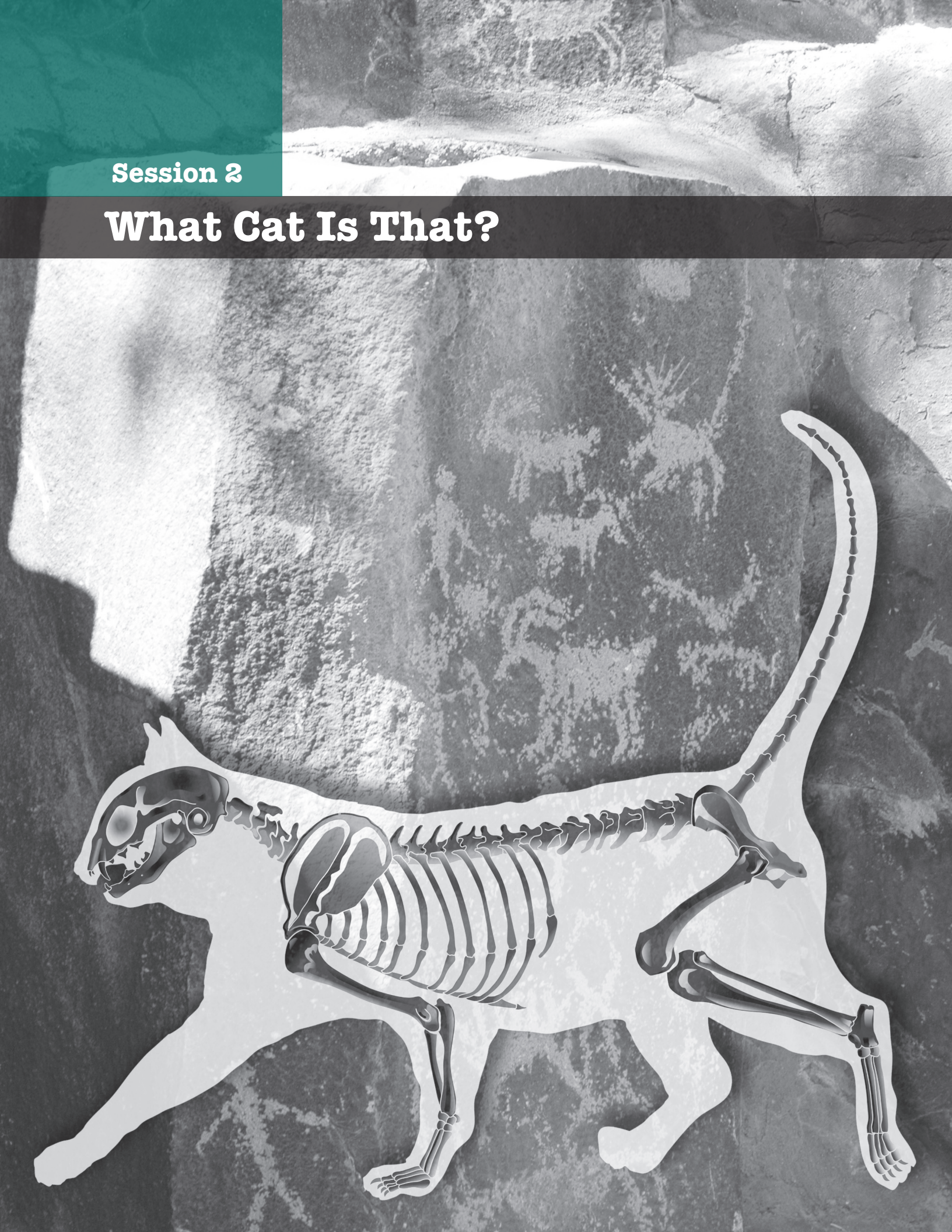
Session 2

What Cat Is That?



Acknowledgments

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Session 2

What Cat Is That?

Objectives

1. Participants will deepen their understanding of prerequisite knowledge for proportional reasoning and related concepts.
2. Participants will identify methods students may use to determine whether organisms could have lived in specific regions and why they became extinct.
3. Participants will be able to reproduce forced perspective images and understand how they may be used to exaggerate information.

Facilities

- A room with broadband Internet access, a data projector, tables, and space for participants to work comfortably in small groups
- Electricity for participants' computers

Equipment/Materials

- Computer with Internet access for facilitator
- Laptop computers with Internet access (1 per group or pair)
- What Cat Is That? PowerPoint
- Hard copies of PowerPoint thumbnails (1 set per participant)
- Calculators (1 per group)
- Chart paper, chart stand, and markers
- Sets of Prerequisite Knowledge Cutouts (see Handout 8 for template) printed on colored card stock, cut into individual items, and placed in resealable storage bags (1 set per pair or group)
- Sets of four resealable storage bags each containing a different type of bean, preferably pinto beans, lima beans, black beans, and red beans (1 set per group)
- Cameras with a wide-angle lens (1 per group)

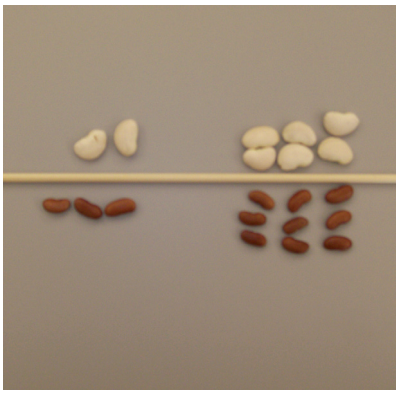
Participants
Up to 25 teachers

Time Required
6 hours

Handouts

- ❑ 1: Prehistoric Cat Activity
- ❑ 2: Prehistoric Cat Solution
- ❑ 3: Knowledge Package: Proportional Reasoning
- ❑ 4: Proportional Reasoning: Fundamental Big Ideas
- ❑ 5: The Meaning of a Fraction Such as 3/4
- ❑ 6: Interpreting the Symbolism
- ❑ 7: A Closer Look at Multiplication
- ❑ 8: Proportional Reasoning Prerequisite Knowledge Cutouts
- ❑ 9: Ancient Cats in Central Texas?

- Tripods for cameras (1 per group, if available)
- Wooden dowels to serve as physical fraction bars (1 per group)
- Connecting Kids Video: Visualizing Proportionality (http://youtu.be/sswKF5AzJVw)



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Software

- Microsoft PowerPoint
- Internet browser

Facilitator Preparation

- Read the session guide and familiarize yourself with the activities and handouts 1–9.
- Preview the What Cat Is That? PowerPoint.
- Make copies of the handouts and PowerPoint thumbnails.
- Preview all the websites used in the session (Handout 9) to ensure the links are current. Bookmark each site.
- Ensure adequate numbers of materials for all participants and groups.
- Prepare the sets of Prerequisite Knowledge Cutouts and beans as described under "Equipment/Materials."
- Solve the mathematics problems to be confident in the processes and solutions and prepared for questions or alternative approaches.
- Review the BSCS 5Es Instructional Model (see Session 1: Handout 1).
- Review the related Texas Essential Knowledge and Skills (TEKS) Mathematics, Science, and Technology Applications standards listed at the end of this session.

Prerequisite Skills of Participants and Facilitator

- Basic computer and data projector skills
- Basic understanding of Web navigation
- Foundational understanding of mathematical topics: equivalent fractions, ratios, change, and proportional reasoning

Grouping Strategy

A heterogeneous grouping strategy should be used. Ensure that each group of two to four members has at least one member with middle school experience or expertise.

Session Sequence

Working primarily in heterogeneous groups, participants will be challenged with a variety of engaging tasks with a focus on the fundamentals of proportional reasoning. The activity framework and process will be facilitated via a PowerPoint presentation and facilitator notes.

Engage

Equipment/Materials

- ❑ Computer and data projector for facilitator
- ❑ What Cat Is That? PowerPoint
- ❑ Hard copies of PowerPoint thumbnails (1 set per participant)

Whole Group

1. Provide each participant with a thumbnail copy of the PowerPoint. Ask that participants do not look ahead to the upcoming slides.
2. Start the PowerPoint and share the following information as you move through the first four slides:
   - **Slide 2:** *"For thousands of years cats have intrigued people. Perhaps it is their aloof manner, the way they slink by when they walk, or the mysterious look in their eyes. For whatever reason, many people find cats fascinating."*
   - **Slide 3:** *"In this activity, we will be looking at a specific cat. This cat's remains were found in a cave near San Antonio, Texas. The remains were identified as being a cat by the femur bone."*
   - **Slide 4:** *"But unlike a domestic cat, which has a femur of about 3 inches in length, the femur of this cat was about 10 inches long. One more piece of evidence is that scientists believe this bone to be approximately 10,000 to 12,000 years old."*

Explore

Equipment/Materials

- ❑ Computer and data projector for facilitator
- ❑ What Cat Is That? PowerPoint
- ❑ Calculators (1 per group)
- ❑ Laptops with Internet access (1–2 per group)
- ❑ Handout 1: Prehistoric Cat Activity (1 per participant)

Table Groups

3. Organize participants into heterogeneous groups of two to four members. Ensure that each group has at least one member with middle school math experience or expertise. Provide each group with a calculator and a laptop computer with Internet access.

4. **Slide 5:** Explain to participants: *"While we might have some ideas as to what kind of cat this was, we need to use science and math to support our hypothesis. For this next activity, we will assume the musculature of the ancient cat and of modern domestic cats are similar. Remember that the fossil femur that was found was about 10 inches long. We also know that the average modern domestic cat weighs about 10 pounds. We will use this information to try to figure out how much the ancient cat might have weighed and its height from the floor to the top of the shoulders when standing on all four paws."*

5. Give each participant a copy of Handout 1: Prehistoric Cat Activity. Paraphrase the instructions on the slide. Allow groups to use calculators and provide 4–5 minutes for small group discussion. Announce that each group will be asked to share its process, reasoning, and results.

Handout 1: Prehistoric Cat Activity

Mickey found the femur (leg bone) of an ancient adult cat that had a build very similar to that of a modern domestic cat. The femur was 10 inches long. Mickey knows that the same bone in an adult modern domestic cat is about 3 inches long. He wants to study the relative bone structure and energy requirements of these ancient cats. To do this, Mickey needs to estimate the relative size of this particular ancient cat. In proportional terms, how does the relative size of the ancient cat compare to the size of a modern domestic cat? As a group, please prepare to explain your solution, approach, and reasoning.

Explain

Equipment/Materials

- ❑ Computer and data projector for facilitator
- ❑ What Cat Is That? PowerPoint
- ❑ Calculators (1 per group)
- ❑ Laptops with Internet access (1–2 per group)
- ❑ Handout 2: Prehistoric Cat Solution (1 per participant)

Whole Group

6. **Slide 6:** Ask a representative from each group to explain the group's solution to the problem as well as the supporting approach and reasoning. Next, give each participant a copy of Handout 2: Prehistoric Cat Solution. Have participants read and discuss the solution.

   As a facilitator, listen for and reinforce the concept that the change is three-dimensional. This fact may surprise many people. If more clarification is needed, use the simpler example of a 1x1x1 cube being changed to a 3x3x3 cube. The new volume is 27 cubic units as compared to the original 1 cubic unit.

7. Add, *"Students often receive limited practice in two-dimensional and three-dimensional change. We as teachers need to provide more experiences that focus on these concepts, especially at the middle school level."*

   Ask participants for examples of how they are teaching this concept in their classrooms. Afterward, share some additional ways to introduce and reinforce two-dimensional and three-dimensional change. Explain that participants will explore this topic, as well as why animals become extinct, in this session.

Elaborate

Table Groups or Pairs

8. **Slide 6** (cont.): Read the slide's second statement about prerequisite knowledge. With participants working in their same groups or in pairs, have them use Handout 2 to list the prerequisite knowledge and skills students need to understand proportional relationships. Tell participants to keep the lists for future reference.

9. **Slide 7:** Ask the groups or pairs to examine the TEKS on the Texas Education Agency (TEA) website to see where proportional reasoning is addressed in K–8 mathematics instruction. Allow 5–10 minutes for research. Participants should discover that ratios and the use of proportional thinking and problem solving are not directly addressed in the K–5 TEKS but are covered in several objectives in grades 6–8. Ask volunteers to share their results.

10. **Slide 8:** Give each group or pair one set of Prerequisite Knowledge Cutouts (see Handout 8 for the template) and ask them to follow the directions on the slide. These directions are open-ended to allow groups the freedom to sort the topics based on their own criteria. Have the groups or pairs share their frameworks and criteria with the whole group.

    Explain that there are no right or wrong answers. The key point is that proportional reasoning is a complex topic that requires prerequisite knowledge and skills.

11. Have the groups or pairs review the lists of prerequisites they generated earlier (Step 8) and compare them with the topics on the cutouts. Depending on time constraints, facilitate a deeper conversation regarding the knowledge necessary for students to think proportionally.

Whole Group

12. **Slide 9:** Explain that a "knowledge package" is a method of listing and organizing facts, skills, concepts, and other knowledge a teacher considers to be necessary for students to learn a new topic. Another name for a knowledge package is a concept map.

13. Give each participant a copy of Handout 3: Knowledge Package: Proportional Reasoning. Explain that the handout presents two possible knowledge packages for proportional reasoning prerequisites, one on each side, and that the two versions differ primarily in how the skill sets are organized. Both were developed at SEDL and are based on the work of Liping Ma (*Knowing and Teaching Elementary Mathematics,* 1999). Again, there are no right or wrong solutions or versions. Facilitate a whole-group discussion of the handout.

Equipment/Materials

- ❑ Computer and data projector for facilitator
- ❑ What Cat Is That? PowerPoint
- ❑ Calculators (1 per group)
- ❑ Laptops with Internet access (1–2 per group)
- ❑ Sets of Prerequisite Knowledge Cutouts (from Handout 8; 1 set per pair/group)
- ❑ Sets of four re-sealable storage bags each containing a different type of bean, preferably pinto beans, lima beans, black beans, red beans (1 set per group)
- ❑ Wooden dowels to serve as physical fraction bars (1 per group)
- ❑ Connecting Kids Video: Visualizing Proportionality (http://youtu.be/sswKF5AzJVw)
- ❑ Chart paper, chart stand, and markers
- ❑ Cameras with a wide-angle lens (1 per group)
- ❑ Tripods for camera (1 per group, if available)

Handouts (1 per participant)

- ❑ 2: Prehistoric Cat Solution
- ❑ 3: Knowledge Package: Proportional Reasoning
- ❑ 4: Proportional Reasoning: Fundamental Big Ideas
- ❑ 5: The Meaning of a Fraction Such as 3/4
- ❑ 6: Interpreting the Symbolism
- ❑ 7: A Closer Look at Multiplication
- ❑ 9: Ancient Cats in Central Texas? (electronic)

14. Give each participant a copy of Handout 4: Proportional Reasoning: Fundamental Big Ideas. Have different teachers read items I–V. The second list of items is a continuation of Item V and shows from the bottom up the essential knowledge students must have to understand each concept in turn. For example, for students to understand three-dimensional change, they must have a strong foundation in two-dimensional change. In turn, for students to understand two-dimensional change, they must have a strong foundation in one-dimensional change. And for students to understand one-dimensional change, they must have a strong foundation in ratios, multiplicative comparison, and other concepts and skills, as illustrated in the knowledge package activity.

15. **Slides 10–12:** These slides focus on mathematical proficiency, which refers to the types of understanding teachers should instill in students. Show Slide 10 and explain the idea of mathematical proficiency. Tell participants that this term was coined in the 2001 National Resource Council report *Adding It Up: Helping Children Learn Mathematics.* This report describes five interwoven competencies that make up mathematical proficiency.

16. **Slides 11–12:** Slides 11 and 12 list the five key competencies. Before the session, become familiar with these five competencies so that you can paraphrase them rather than reading them from the slides. Another option is to have different participants read each competency aloud. Facilitate discussion.

17. **Slide 13:** Focus on the first competency—conceptual understanding—and how critical it is in mathematics education. Ask the participants what they think conceptual understanding "looks like." Have volunteers share their ideas.

18. **Slide 14:** Explain that the information on this slide provides one version of what conceptual understanding looks like. To add variety, have different teachers read individual items.

19. **Slide 15:** Further describe conceptual understanding by using an analogy to the movie *Field of Dreams (*1989), starring Kevin Costner ("If you build it, he will come."). Paraphrase the contents of the slide.

    Add, *"Conceptual understanding truly is the foundation for mathematics and science, and the quality of instruction is dependent on teachers' content knowledge. As teachers, we are continually looking for new activities for our classrooms. However, as the* Field of Dreams *analogy states, if we build our content expertise, the ideas for new activities and lessons will come. Teachers who have deep content understanding find that they are more creative because of their deeper knowledge."*

20. **Slide 16:** Explain that there is a lot of power in conceptual understanding. Show Slide 16 and paraphrase what conceptual understanding enables teachers and students to do. Add that these benefits do not occur in isolation but rather often overlap and connect.

    As a transition, focus on the third benefit of conceptual understanding, which involves language. Say, *"Mathematics is much more than numbers. One of the key factors often overlooked is the idea of language in mathematics. Language plays a key role because it is an integral part of what we teach (e.g., the descriptions and definitions) and how we teach it—the language of instruction."*

21. **Slide 17:** State that conceptual understanding and language support and reinforce each other.
    - Read the first bulleted statement on the slide to reiterate this point. This statement is repeated from the previous slide because it is so important.
    - Read the second bulleted statement. Add, *"Definitions do not have to be complex—quite the contrary, a conceptual definition can be simple yet deep."*

- Read the third bulleted statement. Add, *"An understanding of the definitions and symbolism will naturally influence the type of questions teachers ask students."*
- Continue by saying, *"You probably know a number of effective strategies for teaching mathematics."* Depending on time constraints, ask participants to share some of the strategies they either use or know.

22. **Slides 18–19:** State that this and the next slide list suggested strategies—some of which may be unfamiliar—for helping students develop a deep conceptual understanding of mathematics. Read the strategies aloud and provide examples if necessary. Do the same with the strategies on Slide 19.

23. **Slide 20:** Paraphrase the information. Then add, *"These strategies become more 'natural' with deeper content knowledge."* Note that time constraints do not allow for a detailed explanation of the strategies, but urge participants to consider implementing those strategies that they are not currently utilizing.

Table Groups

24. **Slide 21:** Shift the focus to the critical topic of fractions because ratios, as a subset of fractions, are the critical foundation for proportional reasoning.

    Show Slide 21. State that students must first see and understand the big picture of fractions. Have teachers work in heterogeneous groups of three to four members to develop a tree diagram that illustrates the different possible meanings of the fraction 3/4. Then ask a representative from each group to share the results with the whole group.

25. Give each participant a copy of Handout 5: The Meaning of a Fraction Such as 3/4, which illustrates the possible meanings of 3/4. Have the groups study this version of a tree diagram of the big picture of fractions. Then ask for comments.

26. **Slide 22:** Ask the question stated on the slide. Give participants time to provide answers.

    Pause, then announce that the groups are going to do an activity that models a hands-on approach for establishing the fundamental idea of a ratio without using the typical terminology of fractions. Explain that the activity involves the use of manipulatives, and give each group one wooden dowel and one set of four bags containing different types of beans. Tell participants that the different beans will represent different numbers and the dowel will function as a real-life "fraction bar" by separating beans above and below it. Have the groups use the beans and dowels to create different ratios, such as two red beans above the dowel and five pinto beans below the dowel to represent the ratio 2/5.

27. Ask the groups how they can extend this idea to create equivalent fractions. Give them time to take on this challenge. As an example, on the leftmost part of the dowel, a group might place two pinto beans above the dowel and three lima beans below the dowel, creating a relationship of two pinto beans for every three lima beans (a 2/3 ratio). On the right side of the dowel, the group might then place four pinto beans above and six lima beans below the dowel, creating an equivalent ratio of 4/6. Give the groups time to devise and share examples.

28. As closure, point out that the key for equivalent fractions is to establish a relationship and then to maintain that relationship. Note that in early elementary classrooms, students could create equivalent fractions but not use the related academic language (such as numerator, denominator, and fraction bar). Thus, without realizing they are doing equivalent fractions, students could establish the foundation for ratios and proportional reasoning. Ask for comments.

Inform participants that a short video is available that illustrates the dowel-and-beans activity for establishing the idea of ratios and proportionality. You can access the Connecting Kids Video: Visualizing Proportionality on the SEDL YouTube channel at http://youtu.be/sswKF5AzJVw.

Individuals/Pairs or Table Groups

29. **Slide 23:** Explain that part of the difficulty of fractions is embedded in the symbolism. The task on the slide begins an examination of the problems with the symbolism of fractions. Give each participant a copy of Handout 6: Interpreting the Symbolism, which includes the problem on the slide as well as three of the key ideas that were on the cards used in the knowledge package activity.

    Have participants do the task individually and then share and compare their results with a partner or small group. Ask if anyone considers the equation to be false. Then ask participants to imagine that they had given this task to their class, and lo and behold . . .

30. **Slide 24:** State that this slide illustrates a scenario in which a student states that the shaded area in the left-hand diagram represents 1/2 of a pizza and the shaded area in the right-hand diagram represents 2/4 of a pizza. Obviously, the two shaded areas are not the same size, so the student claims that 1/2 = 2/4 is a false statement.

    Ask participants, *"What is the cause of the confusion in this scenario?"* Someone should explain that the student does not understand that the fractions refer to the same-sized whole. Respond, *"Where in the equation 1/2 = 2/4 does it ensure that the two fractions do in fact refer to the same-sized whole? Herein lies a huge problem: We assume that each fraction refers to the same-sized whole, and we must ensure that students are aware of this assumption."*

31. **Slide 25:** Announce that the plot thickens. Another student also claims that 1/2 = 2/4 is a false statement but use a different approach to justify the claim. Ask participants to examine the slide and volunteer what they think the student's justification is.

    Someone should explain that the student's justification is the idea that the equal sign says that the quantity on one side of an equation is the same as that on the other side. The image shows different quantities on each side of the equal sign, so the student has a point in claiming that the statement is false. (Also, one could use the idea of a balance scale and note that the amount on each side of the fulcrum is different, thus making the scale imbalanced.)

32. Ask, *"What is the fundamental problem in this context?"* Someone should respond that the problem is with the definition of the equal sign. The equation 1/2 = 2/4 is a proportion; thus, the statement is not about quantities, but relationships. The relationship on the left of the equal sign is equivalent to the one on the right. In a sense, a proportion does not adhere to the typical definition of the equal sign because, again, the focus is on relationships and not quantities. This distinction must be made clear to students.

33. **Slide 26:** Explain that this context illustrates confusion with fractions as a result of the symbolism and its interpretation. Have participants do both tasks individually and then share and discuss their results with a partner or small group. After sufficient time, ask for comments or explanations.

    Note that many students will struggle to make sense of this problem if taught to interpret the symbolism as, How many times does 1/2 go into 10? Rather, in most contexts, the question should be, How many halves are in 10 wholes? Students can more easily envision 10 pizzas cut in half. The

question would then be, How many half-pizzas are in 10 pizzas? (20 half-pizzas). The key learning is the importance of the language used in interpreting the symbolism.

34. **Slide 27:** State that the slide illustrates additional confusion students can have with fractions because of the symbolism and its interpretation. As before, have participants do the tasks individually and then share and discuss their results with a partner or group.

    The key points to note are as follows:

    - **Task 1:** Most students would not interpret a fraction in that way because even with an expression such as 3 x 2, which involves only whole numbers, students are not taught to interpret "x 2" as a noun. In essence, if 3 x 2 was interpreted as "3 twos," the expression 3 x 2 could be analogous to 3 cows or 3 pencils. As another example, 23 x 47 is 23 groups or sets of 47 each (or 23 forty-sevens).
    - **Task 2:** Students often have trouble interpreting mixed numbers, probably because some of the symbolism differs from what students are accustomed to seeing with whole numbers. For example, there is no place value for the 1/2 in 3 1/2 as there is for the 2 in 32. Also, 3*y* represents multiplication, but 3 1/2 represents the sum of 3 + 1/2. Further, students may need more practice in the use of the distributive property as shown on the slide.

35. **Slide 28:** State that the investigation of problems due to symbolism and its interpretation continues. As before, have participants do the task individually and then share and discuss their results with a partner or small group. After sufficient time, ask for comments or explanations.

    The key points to note are as follows:

    - First, it is critical that students know the definition of a proportion. A proportion is an equation that states that two ratios are equivalent and express the same relationship.
    - Second, the expression using the division symbol can be rephrased as the proportion 5/4 = Y/1. Students are often asked to blindly perform division with no knowledge as to the purpose or utility of the task. In this scenario, we have a ratio of 5 to 4. However, U.S. society prefers to make comparisons to one. For example, we do not state that we make $60 every 4 hours. We prefer to say that we make $15 per hour. In this example, the quotient is 1.25, but we forget to tell students that this is in comparison to one (5/4 = 1.25/1)!

36. **Slide 29:** State that problems with the symbolism of fractions and its interpretation are compounded by confusion over the idea of the whole. As before, have participants do the task on the slide individually and then share and discuss their results with a partner or small group. After sufficient time, ask for comments or explanations.

    The key points to note are as follows: The 1/2 in the problem uses 1 gallon as the whole. However, the whole for the 2/3 is 1/2 gallon. When the computation is done, the whole for the answer of 1/3 reverts back to 1 gallon! As teachers, we often neglect to point out this aspect of fractions to students—an integral component of a fraction that represents a quantity is the whole associated with that fraction.

37. **Slide 30:** State that problems with symbolism are also compounded by careless or false statements. As before, have participants do the task individually and then share and discuss their answers with a partner or small group. After sufficient time, ask for comments or explanations.

    The key points to note are as follows: The statement "1 inch = 20 miles" is obviously false! However, we often look the other way when maps or drawings make such a claim. To help students avoid confusion, we must attend to precision. What really is happening on a map or drawing is that we are

making a scale model of something in real life. The reality is that we have established a ratio; in this case, 20 miles in real life is being represented by 1 inch on a map.

38. **Slide 31:** Explain that the focus now shifts to the knowledge students need for a solid foundation in proportional reasoning. Give each participant a copy of Handout 7: A Closer Look at Multiplication, which presents the tasks on the slide and also provides room for work and notes. Have participants do the tasks individually and then share and compare their results with a partner or small group. Ask volunteers to report out.

    The key points to note are as follows:

    - Task 1: Most teachers define multiplication as repeated addition, but this definition falls short. The key missing component is that multiplication is a faster way of doing addition through the use of equal-sized groups. This critical idea of equal-sized groups should not only be included, but also be the focal point of the definition, because it enables students to make connections to related topics such as division.
    - Task 2: In most basic contexts, multiplication and division involve the same three components: (a) the number of groups, (b) the equal size of the groups, and (c) the total. The only difference between the two processes is which components are known and which is unknown and must be solved.
    - Task 3: Multiplication involves finding a total based on having so many groups of a certain equal size. An important condition in this context is that the total remain constant. Given a constant total, if the number of groups increases, then the size of the groups must decrease. For example, 24 can be 8 groups of 3. If we increase the number of groups to 12, then the size of each group must decrease as well to 2. If we then decrease the number of groups to 6, the size of each group must increase to 4. Thus, an inverse relationship exists between the number of groups (or sets) and the size of each group (or set).

39. **Slide 32:** Explain that this task is an example of an activity that can help students adopt a multiplicative perspective when comparing quantities. Give each participant blank paper on which to draw the vertical bar graphs. Have participants do the task individually and then share and compare their results with a partner or small group. Ask volunteers to use markers to draw their examples on chart paper on a stand for the whole group to see.

40. **Slide 33:** Explain that this slide shows three possible examples of "5 more." Ask participants, *"What is the important realization that these examples illustrate?"*

    The key learning is that when comparing absolute (or actual) differences, such as "5 more," the results can look very different from one context to another. For example, 6 compared to 1 looks quite different from 10,005 compared to 10,000.

41. **Slide 34:** Explain that this activity is a continuation of the previous one. Have participants do the task individually and then share and compare their results with a partner or small group. Ask volunteers to use markers to draw their examples on chart paper on a stand for the whole group to see.

42. **Slide 35:** Explain that this slide shows three possible examples of "half as much." Ask participants, *"What is the important realization that these examples illustrate?"*

    The examples show that when comparing proportional (or multiplicative) differences, such as "half as much," the results look the same regardless of the size of the quantities. Thus, the proportional comparison has a huge advantage over the absolute comparison, where the relationship differs depending on the quantities involved. The key learning is that teachers must do more of these types of activities to help students make comparisons using both absolute and proportional (multiplicative) perspectives.

43. **Slide 36:** Explain that this task is an example of a problem that can help students transition to proportional reasoning. The problem is also an example of using ambiguity as an asset in instruction. Have participants solve the problem individually and then share and compare their results with a partner or small group. Ask volunteers to report out.

    The key points to note are as follows: The question on the slide is deliberately ambiguous so that teachers (and students) can consider the problem from two perspectives. From an absolute perspective, the larger pig grew more because it grew by 8 pounds, while the small pig grew by only 5 pounds. From a multiplicative perspective (using proportional reasoning), the smaller pig grew more because it doubled in weight (from 5 to 10 pounds, a 100% increase), whereas the larger pig grew by only 8% (from 100 to 108 pounds). It is critical that middle school students make the transition to viewing change from a proportional (multiplicative) perspective.

44. **Slide 37:** Explain that one way of really getting a handle on how well students understand a topic is to ask them to define it. Add that this "definition strategy" can be enhanced by giving students the three options shown on the slide. Note that a student should use only one of the three options. The strongest indicator of student understanding would be a solid response using Option 2, because expressing an idea in one's own words presents a clear picture of one's level of understanding. However, it is important (for standardized tests, textbook content, future math courses, and so on) that students also know a more formal definition (Option 1) that includes the appropriate academic language. A weaker level of knowledge would be indicated if a student must resort to a picture or example (Option 3) to provide a definition.

45. **Slide 38:** Explain that this slide provides an example of the three-option definition strategy. Add, *"This task enables you to use the strategy in the role of students."* Have participants work individually to provide a definition for each option and then share and compare their results with a partner or group. Ask volunteers to report out.

    The key ideas that should emerge are as follows: If students define π as just a number (e.g., 3.14 or 22/7), then they do not understand the concept deeply. The definition should focus on the idea of a relationship, which in turn connects to the idea of proportionality. As a concept, π is simply the ratio of the circumference of a circle to its diameter. Armed with this knowledge, students can do much more with problems or contexts that involve circles or circular distance.

46. Close the math portion of the session by leading a discussion of what participants learned, especially any "aha" moments regarding the mathematics content. Ask that participants consider ways in which the various tasks and activities could be utilized in their specific classrooms with their own students. Since the TEKS do not address proportional reasoning in the earlier grades, possibly discuss why it is important to start laying the foundations for this key concept at an early age.

    This activity illustrates a way to cover equivalent fractions through a ratio/proportion context. Ask participants to reflect on the types of questions (e.g., Bloom's Taxonomy: justify and explain the solution process) they would ask when teaching these math concepts. In addition, discuss strategies for addressing the needs of diverse learners (i.e., making modifications for English language learners or students with disabilities).

Table Groups

47. Organize participants into heterogeneous groups of three to four. Ensure that each group has a laptop with Internet access.

48. **Slide 39:** Tell participants they are going to revisit the Prehistoric Cat activity from the start of the session. This time, however, they will play the role of students so they can apply what they have learned about ratios and proportions. Review the facts about the prehistoric cat problem that are listed on the slide.

49. **Slide 40:** This slide shows an image of a fossilized skeleton of the same type of ancient cat as the one that once roamed Central Texas and whose femur bone was found there.

50. **Slide 41:** Have participants work in their groups to solve the problem presented on the slide. The average modern domestic cat has a 3-inch long femur. The unknown cat femur is 10 inches long. Assuming that the build and musculature of the ancient cat is similar to that of a modern domestic cat, what would be the ancient cat's approximate weight?

    Ask the groups to report their answers. Using the information from the same activity done previously, the ancient cat would be about 37 times heavier than a modern domestic cat. Thus, the estimated weight of the ancient cat would be about 370 pounds (10 x 37).

51. **Slide 42:** A multitude of factors could have contributed to the demise of these ancient cats. This slide lists some possibilities for discussion. (With students, it may be necessary to define the terms *dietary*, *capacity*, and *density*.) Encourage participants to add their own ideas. Then brainstorm some other environmental factors that should be considered.

52. **Slide 43:** Give each participant a copy of Handout 9: Ancient Cats in Central Texas? in electronic form, if possible. Students need to learn to conduct research on the Internet to develop accurate answers to questions such as those on the handout. Ask the groups to assume the role of students and use the websites listed to answer the questions. Tell the groups they have 30 minutes.

Handout 9 Questions:

1. What kind of ancient cat might have existed 10,000 to 12,000 years ago?
2. The fossil femur was found in Central Texas. Is this find plausible?
3. If the ancient cat did live in Central Texas, what would it have needed to survive? Consider both dietary needs and biome.
4. How many of these felines would you predict lived in one area?
5. Why do you think this type of feline disappeared?

After 30 minutes, ask a spokesperson for each group to report what members learned. The prehistoric cat is a Smilodon (saber-toothed cat).

53. **Slide 44:** Explain that the focus now shifts to forced perspective. Tell participants, *"Forced perspective is an optical illusion that makes things seem bigger or smaller than they actually are. In this part of the session, you will create images that illustrate 'impossible' situations without the use of software."*

    Allow 10 minutes for the groups to answer the following questions about forced perspective. The groups may use Internet resources. Afterward, discuss what was learned and add the following ideas if they are not shared by the groups.

- *Who* – people trying to create optical illusions
- *What* – images with objects that appear larger or smaller than in reality
- *Where* – in photographs and often on Internet resources
- *Why* – multiple reasons, including fantasy stories, movies, the desire to exaggerate information
- *How* – with cameras and practice

Whole Group

54. Continue the discussion by asking how forced perspective relates to the fossil femur found in Central Texas. The logic used in this activity was that the proportions of a domestic cat were similar to those of an ancient cat, in this case the Smilodon (or saber-toothed cat). Discuss the following:
    - What do we, as adults, reason could be a difference in the musculature between the two cats? (We can hypothesize that Smilodons should have had heavier musculature.)
    - Why? (Its activities for survival should have been greater.)
    - Why would we use this activity with students? (to make the activity relate to the students' personal experiences)
    - What limitations would we need to discuss with the students? (the difference between anticipated musculature and what the Internet may provide in images with a distorted perspective that should be evaluated carefully)

55. Tell participants: *"Remember that scale is directly related to proportional reasoning. In mathematics, the idea of scale is pretty straightforward, but there are instances where the scale or relationship between or among items is not truly represented. With technology, it is easy to distort the relative scale of two or more visual images. Even without the use of software, situations can be manipulated to create images that can fool the human eye. It is important for students to understand that despite the real mathematical scale or relationship, perspective can be used to distort reality."*

56. **Slide 45:** Tell participants: *"If you have ever seen Cinderella's Castle at Disney World and thought, 'It sure looks smaller than I thought it was,' then you are experiencing forced perspective. By making the windows, doors, and other attributes of the castle much smaller as they occur higher up in the building, the builders played a trick with our visual perception, causing our brains to think things are actually bigger than they are. Forced perspective can also be seen in many tourist photographs. You may have seen a photo of someone 'holding' the moon or 'holding up' the Leaning Tower of Pisa. These images are achieved by the photographer using scaled objects in the shot to manipulate how objects appear."*

57. **Slides 46–48:** Show additional examples.

58. **Slides 49–50:** Say, *"We are going to practice making our own forced perspective images. To create forced perspective images, the photographer needs to have a knowledge of the science of light and of geometry. By opening the iris of the camera less, the photographer can allow more light to strike the image sensor, enabling a deeper depth of field and making objects on separate planes appear in sharp focus. By playing with the geometry, the photographer can change the angles of the shot and the planes on which the objects appear to create images that would otherwise be impossible."*

    Share the tips on the slide as well as these additional tips:
    - Use a tripod.
    - Use a wide-angle lens, if possible.

- Use an f-stop that ensures a high depth of field, like f/16 or f/22.
- Use familiar objects to give a sense of size.
- Experiment and take lots of pictures, making adjustments as you go.

Use the next 30 minutes to let teachers practice making their own forced perspective images.

59. Ask volunteers to share examples of some of the forced perspective photographs they produced. Use the LCD to show the photos to the whole group.
60. Discuss with the whole group how, and in what context, this strategy might be used in classroom instruction.

Evaluate

Whole Group

61. **Reflection.** Lead a discussion about what participants learned and ways they could utilize and apply the information in their specific classrooms considering their individual students and circumstances. Build on the previous reflection that participants did at the end of the mathematics portion of the session.

Ask, *"Which TEKS were addressed during this session?"* Ask participants to be prepared to share by grade level and content area, such as fourth grade mathematics or science. Be sure to discuss how the lesson should be adapted to target the specific expectations at each grade level.

- Science concepts and standards addressed include but are not limited to:
  - *4.10 a* – explore how adaptations enable organisms to survive
  - *5.10 a* – compare structures and functions of species that help them survive in their environment (webbed feet in aquatic animals)
  - *7.11 b* – explain variation within a population or species by comparing external features . . .
  - *7.11 c* – identify changes in genetic traits that have occurred over several generations
  - *8.11 a* – describe producer/consumer, predator/prey and parasite/host relationships as they occur in food webs
- Mathematics concepts and standards addressed
  - *3.2 a* – construct concrete models of fractions
  - *3.2 b* – construct concrete models of equivalent fractions for fractional parts of whole objects
  - *4.2 a* – use concrete objects and pictorial models to generate equivalent fractions
  - *4.2 b* – model fraction quantities greater than one using concrete objects and pictorial models
  - *5.2 a* – generate a fraction equivalent to a given fraction
  - *6.3 a* – use ratios to describe proportional relationships
  - *6.3 b* – represent ratios and percent's with concrete models, fractions and decimals
  - *6.3 c* – use ratios to make predictions in proportional situations
- Discuss the need to make sure instructional time is used as efficiently and effectively as possible to increase student achievement.

62. Discuss strategies for making modifications for English language learners or students with disabilities. This discussion should include how technology was used and how it could help engage all students and facilitate learning.

63. Finally, using the BSCS 5Es Instructional Model handout provided in Session 1, reflect on the BSCS 5Es Instructional Model used in this session and review where each step was implemented:
    - **Engage:** A fossil femur was introduced as a find in Central Texas.
    - **Explore:** Groups were challenged to determine the size of the ancient feline the femur could have belonged to if the proportions were similar to those of a modern domestic feline.
    - **Explain:** Groups explained how they determined the size of the cat.
    - **Elaborate:** Proportional reasoning was discussed and then applied to the possible size of an ancient feline. Then it was determined what type of feline it could have been, its survival needs, and possibly why it became extinct. Forced perspective was used to illustrate differences in proportion and possible applications to increase critical thinking skills.
    - **Evaluate:** Groups were asked to reflect upon and evaluate the application of this session to include appropriate concepts, TEKS, and instructional strategies.

Technical Assistance Follow-Up

The technical assistance provider should set up a time to visit each participant in the classroom during the implementation of one or more concepts covered in this session:

a The reason some animals have become extinct

b. Conceptual understanding of proportions and ratios

c. The use of forced perspective to study ratios or the reasonableness of information, regardless of the source

Texas Essential Knowledge and Skills (TEKS)

Please note, not all grade levels directly relate to this session. Avoid "stretching" the session to make it apply to TEKS other than those listed below. This effort would not be an appropriate use of the students' learning time.

§111.15. Mathematics, Grade 3.

(b) Knowledge and skills.

(2) *Number, operation, and quantitative reasoning.* The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects. The student is expected to:

(A) construct concrete models of fractions;

(D) construct concrete models of equivalent fractions for fractional parts of whole objects.

§111.16. Mathematics, Grade 4.

(b) Knowledge and skills.

(2) *Number, operation, and quantitative reasoning.* The student describes and compares fractional parts of whole objects or sets of objects. The student is expected to:

(A) use concrete objects and pictorial models to generate equivalent fractions;

(B) model fraction quantities greater than one using concrete objects and pictorial models.

§111.17. Mathematics, Grade 5.

(b) Knowledge and skills.

(2) *Number, operation, and quantitative reasoning.* The student uses fractions in problem-solving situations. The student is expected to:

(A) generate a fraction equivalent to a given fraction such as 1/2 and 3/6 or 4/12 and 1/3.

§111.22. Mathematics, Grade 6.

(b) Knowledge and skills.

(3) *Patterns, relationships, and algebraic thinking.* The student solves problems involving direct proportional relationships. The student is expected to:

(A) use ratios to describe proportional situations;

(B) represent ratios and percents with concrete models, fractions, and decimals; and

(C) use ratios to make predictions in proportional situations.

§112.15. Science, Grade 4, Beginning with School Year 2010–2011.

(b) Knowledge and skills.

(10) *Organisms and environments.* The student knows that organisms undergo similar life processes and have structures that help them survive within their environment. The student is expected to:

(A) explore how adaptations enable organisms to survive in their environment such as comparing birds' beaks and leaves on plants.

§112.16. Science, Grade 5, Beginning with School Year 2010–2011.

(b) Knowledge and skills.

(10) *Organisms and environments.* The student knows that organisms undergo similar life processes and have structures that help them survive within their environments. The student is expected to:

(A) compare the structures and functions of different species that help them live and survive such as hooves on prairie animals or webbed feet in aquatic animals.

§112.19. Science, Grade 7, Beginning with School Year 2010–2011.

(b) Knowledge and skills.

(11) *Organisms and environments.* The student knows that populations and species demonstrate variation and inherit many of their unique traits through gradual processes over many generations. The student is expected to:

(A) examine organisms or their structures such as insects or leaves and use dichotomous keys for identification;

(B) explain variation within a population or species by comparing external features, behaviors, or physiology of organisms that enhance their survival such as migration, hibernation, or storage of food in a bulb; and

(C) identify some changes in genetic traits that have occurred over several generations through natural selection and selective breeding such as the Galapagos Medium Ground Finch (*Geospiza fortis*) or domestic animals.

§112.20. Science, Grade 8, Beginning with School Year 2010–2011.

(b) Knowledge and skills.

(11) *Organisms and environments.* The student knows that interdependence occurs among living systems and the environment and that human activities can affect these systems. The student is expected to:

(A) describe producer/consumer, predator/prey, and parasite/host relationships as they occur in food webs within marine, freshwater, and terrestrial ecosystems;

(B) investigate how organisms and populations in an ecosystem depend on and may compete for biotic and abiotic factors such as quantity of light, water, range of temperatures, or soil composition;

(C) explore how short- and long-term environmental changes affect organisms and traits in subsequent populations.

§126.3. Technology Applications, Grades 3–5.

(b) Knowledge and skills.

(1) *Foundations.* The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to:

(A) use technology terminology appropriate to the task;

(B) save and delete files, use menu options and commands, and work with more than one software application.

(2) *Foundations.* The student uses data input skills appropriate to the task. The student is expected to:

(A) use a variety of input devices such as mouse, keyboard, disk drive, modem, voice/sound recorder, scanner, digital video, CD-ROM, or touch screen;

(B) use proper keyboarding techniques such as correct hand and body positions and smooth and rhythmic keystroke patterns;

(C) demonstrate touch keyboarding techniques for operating the alphabetic, numeric, punctuation, and symbol keys as grade-level appropriate;

(D) produce documents at the keyboard, proofread, and correct errors;

(E) use language skills including capitalization, punctuation, spelling, word division, and use of numbers and symbols as grade-level appropriate.

(4) *Information acquisition.* The student uses a variety of strategies to acquire information from electronic resources, with appropriate supervision. The student is expected to:

(A) apply appropriate electronic search strategies in the acquisition of information including keyword and Boolean search strategies; and

(B) select appropriate strategies to navigate and access information on local area networks (LANs) and wide area networks (WANs), including the Internet and intranet, for research and resource sharing.

(5) *Information acquisition.* The student acquires electronic information in a variety of formats, with appropriate supervision. The student is expected to:

(A) acquire information including text, audio, video, and graphics.

(7) *Solving problems.* The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to:

(A) use software programs with audio, video, and graphics to enhance learning experiences;

(B) use appropriate software to express ideas and solve problems including the use of word processing, graphics, databases, spreadsheets, simulations, and multimedia; and

(C) use a variety of data types including text, graphics, digital audio, and video.

(11) *Communication.* The student delivers the product electronically in a variety of media, with appropriate supervision. The student is expected to:

(A) publish information in a variety of media including, but not limited to, printed copy, monitor display, Internet documents, and video; and

(B) use presentation software to communicate with specific audiences.

§126.12. Technology Applications (Computer Literacy), Grades 6–8.

(b) Knowledge and skills.

(2) *Foundations.* The student uses data input skills appropriate to the task. The student is expected to:

(A) demonstrate proficiency in the use of a variety of input devices such as mouse/track pad, keyboard, microphone, digital camera, printer, scanner, disk/disc, modem, CD-ROM, or joystick;

(B) demonstrate keyboarding proficiency in technique and posture while building speed;

(C) use digital keyboarding standards for data input such as one space after punctuation, use of em/en dashes, and smart quotation marks; and

(D) develop strategies for capturing digital files while conserving memory and retaining image quality.

(4) *Information acquisition.* The student uses a variety of strategies to acquire information from electronic resources, with appropriate supervision. The student is expected to:

(A) use strategies to locate and acquire desired information on LANs and WANs, including the Internet, intranet, and collaborative software; and

(B) apply appropriate electronic search strategies in the acquisition of information including keyword and Boolean search strategies.

(5) *Information acquisition.* The student acquires electronic information in a variety of formats, with appropriate supervision. The student is expected to:

(A) identify, create, and use files in various formats such as text, bitmapped/vector graphics, image, video, and audio files.

(7) *Solving problems.* The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to:

(A) plan, create, and edit documents created with a word processor using readable fonts, alignment, page setup, tabs, and ruler settings.

(10) *Communication.* The student formats digital information for appropriate and effective communication. The student is expected to:

(A) use productivity tools to create effective document files for defined audiences such as slide shows, posters, multimedia presentations, newsletters, brochures, or reports;

(B) demonstrate the use of a variety of layouts in a database to communicate information appropriately including horizontal and vertical layouts;

(C) create a variety of spreadsheet layouts containing descriptive labels and page settings;

(D) demonstrate appropriate use of fonts, styles, and sizes, as well as effective use of graphics and page design to effectively communicate.

(11) *Communication.* The student delivers the product electronically in a variety of media, with appropriate supervision. The student is expected to:

(A) publish information in a variety of ways including, but not limited to, printed copy, monitor display, Internet documents, and video;

(B) design and create interdisciplinary multimedia presentations for defined audiences including audio, video, text, and graphics; and

(C) use telecommunication tools for publishing such as Internet browsers, video conferencing, or distance learning.

References

Bybee, R. W., Taylor, J. A., Gardner, A., Van Scotter, P., Carlson Powell, J., Westbrook, A., & Landes, N. (2006). *The BSCS 5E Instructional Model: Origins, effectiveness and applications.* Retrieved from http://www.bscs.org/bscs-5e-instructional-model

Kilpatrick, J., Swafford, J., Findell, B., and Mathematics Learning Study Committee, National Research Council (U.S.). (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, D.C.: National Academy Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahway, NJ: Lawrence Erlbaum Associates.