

# QuickTakes

A publication of the Southwest Consortium for the Improvement of Mathematics and Science Teaching

## Improving Mathematics Education

### Improving Mathematics Education: What Can We Learn from International Studies?



Many mathematics educators have grown weary of studies comparing math teaching and learning in the U.S. and other countries, wondering whether such comparisons offer anything more than fuel for political debates. Given differences in culture, as well as educational structures and practices, it seems foolish to think that foreign methods and curricula can simply be transplanted to American soil. Can we, then, make much practical use of the information in those studies.

The answer is an emphatic *yes*, says Maria Torres, a former principal and math teacher who works with the Southwest Consortium for the Improvement of Mathematics and Science Teaching (SCIMAST). "Studies like the Third International Mathematics and Science Study (TIMSS) aren't prescriptive," she explains. "They don't say, *Take two doses of Japanese teaching methods and call us in the morning*. I think of them as assessment tools, part of a portfolio of knowledge about our instructional strengths and weaknesses."

As an example, Torres points to one of the TIMSS reports, *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*.<sup>1</sup> This component of the TIMSS was an in-depth exploration of eighth-grade math lessons in the U.S. and six other countries, including Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, and Switzerland.

"This video study," says Torres, "takes instruction apart in ways that let us see all the complexities below the surface. And the results are surprising."

The TIMSS assessment found that eighth-graders' math achievement was significantly higher in all six of the other participating countries than in the U.S. However, the video study found that these countries reflect no single curricular emphasis or instructional approach that would set them apart. On the surface, at least, the countries appear as different from each other as from the U.S., and appear to share as many similarities as differences with American classroom structures, curricula, and teaching methods.

Why, then, do U.S. students lag behind? Understanding the possible answers, says Torres, requires a close analysis of teachers' instructional behaviors.

#### A Closer Look at the TIMSS Video Study

For the video study, more than 600 eighth-grade math lessons were videotaped and collected from the participating countries. A team of researchers then analyzed the lessons in great detail, focusing on some 75 different features of the lessons. The researchers even went so far as to break the lessons down to the level of individual math problems; nearly 15,000 separate problems were analyzed and coded. As a result, researchers were able to probe deeply into the mathematics of each lesson and the ways in which math content was developed.

Researchers found that the many different lesson features could be clustered into three broad dimensions of instruction: the way lessons were structured and organized, the mathematical content, and instructional practices, or the ways in which content was worked on during the lesson.

In terms of the *structure of lessons*, researchers found that, in all seven countries, most of the lesson was devoted to solving mathematical problems. Countries varied in their

<sup>1</sup> Hiebert et al., *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: National Center for Education Statistics, 2003. Several resources related to this study can be accessed or ordered via the National Center for Education Statistics web site, at <http://nces.gov/timss>. These include the full report, a highlights brochure, and a set of CD-ROMs featuring some of the videotaped lessons.

emphasis on reviewing vs. introducing new content, in the number of problems worked on during a single lesson, and the proportions of whole-class vs. private work on solving problems.

Looking at the *mathematical content* of the lessons, researchers noted that the problems addressed five broad topics: number, geometry, statistics, algebra, and trigonometry. In all seven countries, more than 80 percent of problems addressed three of these topics: number, geometry, and algebra. In every country except Japan, more than 60 percent of the problems were categorized as being of low procedural complexity, that is, containing no sub-problems or tasks embedded in the larger problem and requiring four or fewer decisions by students. In Japan, in contrast, only 17 percent of problems were of low procedural complexity, and a far greater percentage — 39 percent as opposed to a high of 12 percent for any of the other countries — were of high procedural complexity. Japan was also the only country in which a significant portion of problems included mathematical proofs (26 percent, as opposed to a high of 3 percent in any of the other countries).

In analyzing *instructional practices*, researchers looked at a number of elements, including the level of teacher vs. student talk and the use of such things as real-life contexts, physical materials or visual representations (drawing or diagrams), applications (problems "that ask students to decide how to use procedures rather than just execute them")<sup>2</sup>, and alternative solution methods. In many of these areas, practices in Japan differed significantly from those in other countries.

After looking at these dimensions of instruction separately, researchers then sought to fit the pieces back together into a coherent portrait of instruction in each of the seven countries. Overall they found that

...the countries that show high levels of achievement on TIMSS do not all use teaching methods that combine and emphasize features in the same way. Different methods of mathematics teaching can be associated with high scores on international achievement tests... Japanese eighth-grade mathematics teaching contains a number of distinctive features. Nonetheless, it appears that these features are not a necessary condition for high achievement in other countries.<sup>3</sup>

## What about the United States?

Math lessons in U.S. schools share a number of similarities with features of lessons taught in other, higher achieving countries. Where, then, do the significant differences lie?

Perhaps the most critical difference noted by this study is the extent to which instruction in the United States emphasizes

mathematical procedures rather than the conceptual underpinnings of those procedures. This conclusion was reinforced by a separate analysis in which a small group of mathematicians and post-secondary teachers rated the level of mathematical development in a random subsample of lessons from each country. Ratings ranged from 1 to 5, with 1 indicating "that a lesson was descriptive or routinely algorithmic, with little mathematical justification provided by the teacher or students for why things work as they do." In contrast, a rating of 5 indicated a lesson "in which the concepts and procedures were mathematically motivated, supported, and justified by the teacher or students."<sup>4</sup> Nearly half (40 percent) of the U.S. lessons received a rating of 1, while none of the other six countries received a rating of 1 on more than 15 percent of lessons.

The emphasis on procedural knowledge emerged in teachers' implementation of the lessons, even when the stated purpose of the lesson highlighted conceptual understanding:

The U.S. falls in the middle of the distribution of higher achieving countries with regard to problems posed with an apparent emphasis on applying mathematical procedures versus examining mathematical relationships. But when the problems were worked through with the class, the U.S. moved to the end of the continuum, with little attention paid to the conceptual underpinnings of the mathematics.<sup>5</sup>

Findings from the study suggest that, in many instances, U.S. teachers may think they are developing students' knowledge of mathematical concepts and relationships, but in practice their focus is merely procedural. When interviewed for the study, U.S. teachers "expressed more confidence" than teachers from other countries that their lessons were aligned with current ideas about teaching mathematics.<sup>6</sup> However, nearly 70 percent of the math problems in U.S.-taught lessons were posed with the intent of using procedures. And of the 17 percent of problems whose intent was to make mathematical connections, less than one percent were actually solved "by explicitly and publicly making the connections."<sup>7</sup>

U.S. lessons also tended to focus more heavily on reviewing old content than on introducing new content. As a result, the TIMSS video researchers concluded that U.S. mathematics teaching reinforces "a more limited and thinner range of learning experiences for U.S. students than their peers in higher achieving countries receive."<sup>8</sup>

<sup>2</sup> Ibid., p. 116

<sup>3</sup> Ibid., p. 149.

<sup>4</sup> TIMSS Video Mathematics Research Group, "Understanding and improving mathematics teaching: Highlights from the TIMSS 1999 Video Study," Phi Delta Kappan, June 2003, p. 774.

<sup>5</sup> Ibid.

<sup>6</sup> Hiebert, et al., p. 34.

<sup>7</sup> Ibid., p. 145.

<sup>8</sup> TIMSS Video Mathematics Research Group, p. 774.

## Why Are We So Stuck on Procedures?

In analyzing the data from international studies, researchers have explored the possible reasons why math teaching in the U.S. continues to focus so heavily on isolated procedural knowledge. The answer, they suggest, is cultural.

In discussing a predecessor to the 2003 TIMSS video study, James Stigler and James Hiebert<sup>9</sup> conclude that teaching is a cultural activity which, like other elements of culture, is learned informally through participation in a particular type of activity over long periods of time. In other words, teachers tend to teach in the ways that they themselves were taught. In the United States, the procedural, "just-the-facts, ma'am" approach to math instruction goes back for generations — and it starts as early as kindergarten or first grade. By the time a prospective teacher reaches college, that perspective is deeply rooted. As Stigler and Hiebert characterize it, such deeply ingrained cultural knowledge becomes an internal "script" that shapes both teachers' behavior and their expectations.<sup>10</sup> Their point is affirmed by educational researchers Karen Osterman and Robert Kottkamp:

Educators are the most thoroughly socialized of all professional groups... Through this lengthy and relatively consistent [schooling] experience, complex sets of knowledge, assumptions, beliefs, and behaviors are ingrained in us so deeply that we are no longer aware of them, yet they guide even minute details of our daily work behaviors.<sup>11</sup>

According to Osterman and Kottkamp, there is often a critical distinction between what individuals are consciously aware of thinking and believing — described as our "espoused theories" —and our "theories-in-use," which are often beyond our conscious awareness and which contain the assumptions, beliefs, and habits of mind that most strongly guide our behavior. The authors note that espoused theories "readily incorporate new information" but that theories-in-use are resistant to change; as a result, "while we adopt new ideas, our behavior often continues unchanged."<sup>12</sup>

A related issue, says SCIMAST's Maria Torres, is that, having been taught a predominantly procedural knowledge of mathematics, many U.S. teachers lack some of the deep conceptual understandings they seek to instill in students. As the National Research Council concludes, "Teachers are unlikely to be able to provide an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way."<sup>13</sup>

Does the cultural entrenchment of procedural approaches mean that changing mathematics instruction is an impossible task? Not at all, says Torres. "What's needed, though — along with strengthening teachers' own conceptual understandings — is to bring those taken-for-granted, largely unnoticed habits into conscious awareness. And the best way we've found to facilitate that process is some form of reflective practice."

There are a number of useful reflective practice strategies that teachers can adopt, from teacher study groups and action research to the Lesson Study model. "We've found Lesson Study to be a particularly valuable strategy," Torres explains. Lesson Study is a process widely used in Japan, in which teachers "jointly plan, observe, analyze, and refine actual lessons."<sup>14</sup> Teachers also can learn a lot from reviewing some of the videotaped lessons analyzed in the TIMSS video study.

Whatever strategy teachers use, says Torres, "the key is understanding the importance of reviewing and reflecting on our classroom interactions. Remember the old saying, *There's many a slip between the tongue and the lip?* All too often, there's a similar gap between what we *intend* to teach and what we actually convey in the classroom.

<sup>9</sup> Stigler & Hiebert, *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press, 1999. This book draws on a 1995 TIMSS video study that addressed the countries of Germany, Japan, and the United States.

<sup>10</sup> *Ibid.*, p. 85.

<sup>11</sup> Osterman & Kottkamp, *Reflective practice for educators: Improving schooling through professional development*. Newbury Park, CA: Corwin Press, 1993, p. 7.

<sup>12</sup> *Ibid.*, p. 12.

<sup>13</sup> National Research Council, *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press, 2001.

<sup>14</sup> Lesson Study Group at Mills College web site, [www.lessonresearch.net](http://www.lessonresearch.net).

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## In the Teacher's Lounge

**Pat:** Hey Chris, what's got you looking so grumpy?

**Chris** [*waving some papers in the air*]: Somebody put yet another article in my in-box, telling me how much sorrier I am than teachers in Japan and Europe.

**Pat:** You mean you got your name in print in the Phi Delta Kappan?

**Chris** [*exasperated*]: Of course it's not about me personally. In fact, that's the problem: I don't see me and my classroom reflected in any of these articles.

**Pat:** So what exactly does this one say?

**Chris:** I don't know exactly, I haven't read it yet. But they're all the same: the grass is always greener in the rest of the world.

**Pat:** So do you think the studies are biased, or just poorly done?

**Chris:** Hey, whose side are you on, anyway? You science teachers are getting bashed, too!

**Pat:** I'm not on any side. But I'm a parent as well as a teacher, and frankly I spend a lot of time worrying about the quality of my kids' education. Katie, my oldest, detested math by the time she got to fifth grade, and I consider that a teaching problem. I think a lot, too, about whether I might be one of those teachers that other parents worry about. So I want to know about anything that can help me, or our school system as a whole, do better.

**Chris** [*sinking onto the lounge's ancient, sagging sofa*]: Well of course I worry about all that, too. And I go to all the trainings, try all the new methods. But where do I get credit for working my buns off to do a good job?

**Pat:** Look, just read the article and see if there's anything of use.

[*Chris, sighing, starts to read. After a few minutes, she sets the papers down on top of the sofa's Florida-shaped coffee stain.*]

**Pat:** Well?

**Chris:** Well I guess I have some thinking to do.

**Pat:** About. . . .?

**Chris:** It says here that, even when U.S. teachers think we're using the new methods, we tend to fall back into the same old patterns. We're still not getting to conceptual understanding, and that's what separates us from these other countries.

**Pat:** Is there a particular approach that seems to work the best?

**Chris:** That's the shocker — these other countries are all over the map, if you'll excuse the pun. But no matter what approach they use, they manage to go deep into the concepts.

**Pat:** So what makes the difference?

**Chris** [*jumping up*]: That's what I intend to find out.

**Pat:** Where're you going?

**Chris:** Principal's office. It's time we got started on that Japanese lesson study business.

**Pat** [*with a shake of the head*]: Now that's what I call a 180.

**Chris** [*walking out the door*]: And that's what I call another useful mathematics concept.