Constructing Foundations for Success:
Implications of the National Mathematics Panel Report

Mathematics Scenario

Objective: To illustrate the types and depth of mathematics that students need to learn, and in turn, that educators need to teach.

I) You will be teaching a mathematics unit on multiplying mixed numbers. Part of the planning of a unit or lessons necessitates the consideration of the prerequisite knowledge that the students will need in order to successfully learn this new skill. List the concepts and skills that students must have already mastered in order for them to be successful with this new topic. (multiplication of mixed numbers)

II) Your students have been taught multiplication of mixed numbers using the standard algorithm (convert to improper fractions, multiply, then simplify and convert back to a mixed number if applicable). You are tutoring Johnny because he still “doesn’t get it”. Using 3 1/2 times 2 1/2 as an example, explain how you would go about instilling Johnny with a conceptual understanding of mixed number multiplication using other perspectives or approaches. Add any additional explanation that you feel is warranted.

Multiplying Mixed Numbers—Making Connections

What critical concepts do students need to master to truly understand the multiplication of mixed numbers?

In most cases in American mathematics education, classes are taught procedurally (“how to”) with little attention paid to the “why” behind the solving of a problem. This activity demonstrates how to look conceptually at mathematics problem solving while looking more closely at the interconnectedness among mathematics concepts.

This activity encourages students to look at the multiplication of mixed numbers in various ways and to make connections among those mathematical perspectives.

Ask students to solve the problem 3 1/2 times 2 1/2 the following ways:

1. From the standard algorithm perspective
2. From a geometry/measurement perspective (using area)
3. From a number/operations perspective (using the definition of multiplication)
4. From an algebraic perspective (with the use of the distributive property)

Then, ask students to make connections among the different views above.

Multiple Multiplication Perspectives

1. **The standard algorithm perspective**

   This approach is the most common and dependent on rules and procedures. Students change the mixed numbers to improper fractions, do the necessary multiplication, then simplify the fraction and/or convert the answer to a mixed number.

   3 1/2 x 2 1/2 =

   7/2 x 5/2 = 35/4 = 8 3/4

2. The geometric or measurement perspective using area

Many teachers are familiar with this approach, especially with whole number multiplication. The area model can be adapted fairly easily to fit a simpler mixed number example such as 3 1/2 x 2 1/2. This perspective, like the definition of multiplication perspective, encourages the student to use a visual approach to the problem. In this case, the tool used to solve the problem is a rectangular grid using the idea of area and square units. Drawn to scale, each rectangle reflects the size of the fraction it represents using a square as the unit. Students can solve this without actually doing any "multiplication". It is preferable to do this with a 4 by 3 grid (Figure 1) so that the students can more clearly see the how the fractional area units of 1/2 and especially 1/4 are derived.

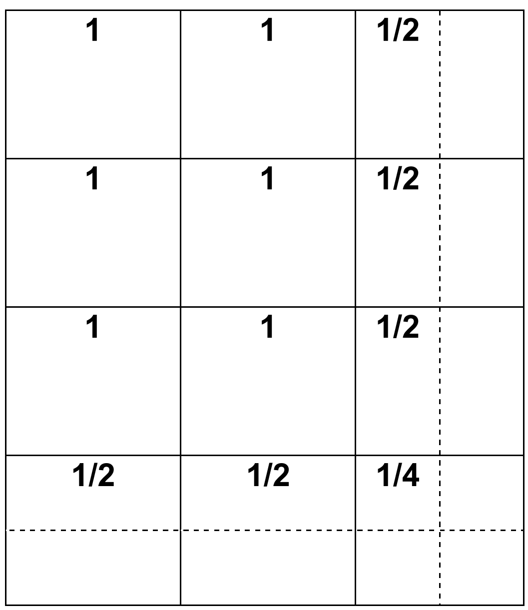

Figure 1

Group the results into similar "units" of 1, 1/2, and 1/4 and combine.

6	(six individual whole units)
2 1/2	(five one-half units – three vertical and two horizontal)
+ 1/4	(one fourth of a whole unit--derived from 1/2 of a half-unit)
8 3/4	sum of the rectangular units, "unit" being a 1 x 1 square.

3. Definition of multiplication

Visualizing the quantity based on what multiplication actually means may enable students to have a deeper understanding of what the multiplication of two mixed numbers entails conceptually. If 3 x 2 by definition is three groups of 2 in each group, then 3 1/2 x 2 1/2 means three and a half groups of two and a half in each group. A pictorial representation (Figure 2) would enable students to come up with the correct solution simply by counting the “wholes” (6), combining the five 1/2s, and the 1/4.

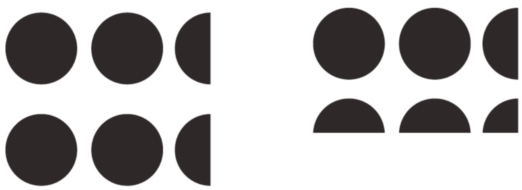

Figure 2

Figure 1 also connects well to the definition of multiplication. If one looks at the picture horizontally, you have the 3 1/2 groups of 2 1/2 each. (And if one looks vertically, you have 2 1/2 groups of 3 1/2 each)

4. Algebraic perspective using the distributive property

Students may be unaware that they have been using the distributive property ever since they started multiplying multidigit numbers. They may need to see whole numbers such as 23 x 54 multiplied horizontally as opposed to the vertical method that is the norm.

23 x 54 = (20 + 3) x (50 + 4)
= 20 x 50 + 20 x 4 + 3 x 50 + 3 x 4
= 1000 + 80 + 150 + 12
= 1242

If they see and understand that process, then students can make the connection and transfer to the multiplication of 3 1/2 and 2 1/2:

(3 + 1/2) (2 + 1/2)
3 x 2 + 3 x 1/2 + 1/2 x 2 +1/2 x 1/2
= 6 + 1 1/2 + 1 + 1/4
= 8 3/4

Making Connections

It is imperative that students be asked to make connections among the different approaches. If students are successful, the outcome should include the following:

If students link the perspectives, the critical discovery is that a common pattern of partial products develops among the last three perspectives. Three and a half groups of two and a half each relates to the layout of the rectangle (area perspective). Each of the first three rows in the rectangular grid consists of 2 1/2 of the square units, while the last row consists of half of what each “normal” row contains. The partial products illustrated in the distributive property perspective can easily be connected to the visual representations in both the area and “definition of multiplication” models.