Classroom Compass

September 2002

HIGHLIGHTS

Getting to the Heart of the (Subject) Matter

If you've tuned in to discussions about reforming mathematics and science instruction, you've probably heard about "teaching for understanding," or "teaching to the big ideas." This issue of the *Compass* explores the meaning behind these catchphrases and the implications for teachers and the educational system that supports — or impedes — teachers' efforts at reform.

One of the foundations of the effort to reform mathematics instruction is the focus on "big ideas" within the specific subject matter: key principles and concepts that underlie particular procedures and formulas, and that link bits of information into coherent patterns of understanding. Both the national standards and reform-based curricula are grounded in the study of major mathematical ideas *and* in teaching these ideas through an active process of exploration and problem-solving.

Yet our experience, as well as much of the research literature, suggests that many teachers in the United States have great difficulty focusing their instruction on major concepts. As a way of informally testing this observation, staff from the Southwest Consortium for the Improvement of Mathematics and Science Teaching (which produces this teacher bulletin) asked middle school mathematics teachers a question: *If your students could learn only one mathematics concept while taking your class, what would you want that concept to be?*

Continued on page 2

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Getting to the Heart of the (Subject) Matter *continued*

We anticipated a number of possible responses — proportional relationships, for instance, or understanding variables. After all, students often have great difficulty with proportional reasoning, and understanding the concept of *variable* "provides the basis for the transition from arithmetic to algebra and is necessary for the meaningful use of all advanced mathematics" (Greenes and Findell 1999).

To our surprise, not a single teacher replied by naming a mathematical concept. Instead, teachers listed a variety of goals unrelated to mathematics content, such as instilling a love of learning, teaching students to think for themselves, or helping students to believe in their own potential.

These are all worthy goals, of course. However, we believe it is no accident that teachers did not address mathematical ideas in spite of being specifically asked to do so.

For decades, American mathematics instruction has been organized around the presentation of specific number facts and procedures. The TIMSS (Third International Mathematics and Science Study) report, which examines instructional practices in the United States and a number of other countries, concludes that "Our curricula, textbooks, and teaching all are a mile wide and an inch deep…Mathematics curricula in the U.S. consistently cover far more topics than is typical in other countries" (McKnight, Raizen, and Schmidt 1999). A videotape comparison of teachers' classroom instruction in the United States, Germany, and Japan reveals that "American teachers…rely more on rote procedures and memorization and are less likely to push students to solve problems or gain understanding of underlying concepts" (Edwards 2000).

Why aren't we teaching for understanding? What's involved in getting to the heart of our subject matter, and what can we learn from other countries that seem to have mastered the task?

"Deep, Vast, and Thorough"

As the Glenn Commission report on mathematics and science teaching points out, "High-quality teaching requires that teachers have a deep knowledge of subject matter. For this there is no substitute" (National Commission on Mathematics and Science Teaching for the 21st Century 2000). Staff from the NSF-funded program, "Teaching to the Big Ideas," concur:

> Teachers' mathematical understanding is central. To achieve a successful practice… teachers require a qualitatively different and significantly richer understanding of mathematics than most now possess (Greenes and Findell 1999).

Such "profound understanding of fundamental mathematics," as researcher Liping Ma calls it, is "deep, vast, and thorough." She explains:

> I define *understanding a topic with depth* as connecting it with more conceptually powerful ideas of the subject. The closer an idea is to the [basic] structure of the discipline, the more powerful it will be, consequently the more topics it will be able to support. *Understanding a topic with breadth,* on the other hand, is to connect it with those of

> similar or less conceptual power. Depth and breadth, however, depend on thoroughness — the capability to 'pass through' all parts of the field — to weave them together again… It is this thoroughness which 'glues' knowledge of mathematics into a coherent whole (1999).

In her research, Ma explored mathematical knowledge among U.S. and Chinese elementary school teachers and found great differences in the depth and range of their understandings. Her work provides concrete illustrations of the ways in which teachers' subject matter understanding influences the ways in which they go about their instructional tasks. (For some highlights, see this newsletter's article, "Know How, and Also Know Why.")

The Elements of Understanding

According to Ma, teaching and learning grounded in profound understanding has four important characteristics. The first is attention to basic ideas, or the "simple but powerful basic concepts and principles of mathematics," such as *equality* or *proportion* (1999). Teachers with deep subject matter knowledge "tend to revisit and reinforce these basic ideas," establishing a solid basis for later learning and reinforcing students' understanding as they move into more complex operations.

The second characteristic Ma identifies is *connectedness* (1999). Teachers are able to understand the links between concepts, to think in terms of what Ma describes as "knowledge packages," that is, "to see mathematical topics group-by-group rather than piece-by-piece." For example, the subtraction problem

$$\begin{array}{r} 52 \\ -\,29 \\ \hline \end{array}$$

involves the concept of subtraction with regrouping. That concept is linked to a number of others, including addition with carrying, composing and decomposing a higher value unit, addition and subtraction within the ranges of 20 and of 10, subtraction without regrouping, the composition of 10, and addition and subtraction as inverse operations. When teachers understand the interconnections among these concepts, they can identify learning sequences that build on students' prior understandings. As a result, Ma observes, "Instead of learning isolated topics, students will learn a unified body of knowledge."

A third characteristic is the capacity to take *multiple perspectives* to find alternate ways of stating a mathematical problem and finding a solution. When teachers — and students — move beyond the narrow focus on a particular procedure, they find that there are a great variety of strategies for solving a specific problem. Such variety reinforces the understanding of mathematics as a dynamic process and echoes the way mathematicians actually work. Ma cites a study in which professional mathematicians were asked to estimate the solutions to multiplication and division problems. The researchers were surprised by the variety of estimation strategies that the mathematicians used. They rarely used the standard algorithm to solve the problems; instead, they used "strategies involving the understanding of arithmetical properties and relationships" (1999).

Having and using conceptual understanding, however, does not mean that mathematicians — or teachers — devalue or ignore procedural knowledge. Among the knowledgeable teachers Ma studied:

> Procedural topics and conceptual topics were interwoven. The teachers who had a conceptual understanding of the topic and intended to promote students' conceptual learning did not ignore procedural knowledge at all. In fact, from their perspective, a conceptual understanding is never separate from the corresponding procedure where understanding "lives" (1999).

The fourth characteristic is what Ma labels *longitudinal coherence*. Teachers who have developed "a profound understanding of fundamental mathematics" are not limited to the knowledge that should be taught in a certain grade." Rather, they know what students are learning in grades below and above their own, and what students need to know to be prepared for each level (1999).

Other experts add another item to the kinds of knowledge teachers need in order to be effective: understanding of the kinds of informal knowledge students bring to mathematics instruction and the way students think as they address problems and concepts:

> It is well known that novices do not think like experts, and, indeed, children are not simply miniature adults; their thinking is qualitatively different than that of adults. In order to help all students achieve mathematical power, teachers need to know a lot about the thinking of students (Bright 1999).

Getting to the Heart of the (Subject) Matter *continued*

A Fragmented System

In this country, most teachers' subject matter understanding appears to fall far short of the characteristics listed in the preceding paragraphs. Ma describes U.S. elementary school teachers' understandings as "fragmented," their computational knowledge generally "limited and flimsy…not solidly supported by conceptual understanding." In many cases, though they were able to calculate the correct response to a problem, U.S. teachers "did not understand why the computational algorithm worked" (1999). Ma notes that:

> The fragmentation of the U.S. teachers' mathematical knowledge coincides with the fragmentation of the mathematics curriculum and teaching in the U.S. found by other researchers as major explanations for unsatisfactory mathematics learning in the U.S.

Middle and high school mathematics teachers — at least those who are actually teaching in their field of certification — tend to know a good deal more mathematics content than elementary school teachers. Though, even at upper levels, teachers' knowledge is often fragmented, focused on discrete facts and procedures. Few teachers at any level have developed systematic understandings of the ways in which students think and learn mathematics.

The most basic problem seems to lie with the mismatch between the kinds of preparation we typically offer teachers in the U.S. and what is required in order to effectively teach for understanding. Most U.S. teachers are not trained to think about mathematics as a system of ideas; they weren't taught that way as children, they weren't taught to teach that way, and the system does not offer opportunities for in-depth learning once they reach the classroom.

Sixth grade Oklahoma mathematics teacher Lesley Zellner speaks about the education gap between studying and teaching mathematics. Zellner, a first year teacher with an elementary education degree, describes her early schooling: "My elementary and secondary math education gave me a love of mathematics. But I feel it left major gaps in my conceptual knowledge (and) emphasized memorization and procedure." Zellner describes her college preparation as "outstanding," as it was "designed to force students to confront foundational mathematics concepts. As I began to consider the best way to relate these concepts to students, I personally made many connections that I had not recognized before." Now that she is teaching, Zellner relies on professional development experiences such as SCIMAST's Summer Academies and teacher education programs to fill in the gaps in her fundamental understanding.

Education Week's year 2000 report on "Quality Counts" reveals that "a substantial chunk of U.S. teachers lack a solid grounding in the subjects they teach." According to data from the National Center on Educational Statistics, 66 percent of high school teachers have a major in an academic field, compared to only 44 percent of middle school teachers and 22 percent of elementary teachers. In addition, more than a fourth of all U.S. teachers begin teaching "without having fully met state licensing standards," and about a third of them "are assigned at least one class a day for which they have not been trained" (Edwards 2000).

These national circumstances are reflected in the five states addressed by this bulletin. Arkansas, Louisiana, New Mexico, Oklahoma and Texas all have requirements for coursework and subject area specialties at the high school level. For middle school teachers, however, only Oklahoma requires a subject area concentration, and coursework requirements differ significantly among the states. Perhaps most significant, none of the five states has specific policies regarding the proportion of time a teacher can be assigned outside their certification area without a waiver, and all states have loopholes that allow districts to circumvent subject area requirements.

Strengthening Teachers' Subject Matter Knowledge

Liping Ma identifies three different periods during which teachers' mathematical understandings can be nurtured: during their own schooling as children, during teacher preparation programs, and during their teaching careers. She observes that:

> In China, the cycle spirals upward. When teachers are still students, they attain mathematical competence. During teacher education programs, their mathematical competence starts to be connected to a primary concern about teaching and learning. Finally, during their teaching careers…they develop a teacher's subject matter knowledge (1999).

In the U.S., most teachers have not had the benefit of this upward spiral — though there is greater hope for tomorrow's teachers as reform efforts begin to influence each of these levels. Even so, many things can be done to build understandings among current and prospective teachers.

Certainly the problems described here have implications for the ways future teachers are prepared. Some research studies indicate that teachers' certification in the subject they are teaching, combined with an academic major in that subject, are correlated strongly with improved student performance (1999). However, a number of studies specific to mathematics have found little correlation between "the number and type of mathematics courses taken or degrees obtained" and student learning.

Ma and others believe that teachers deepen their mathematical understanding after they enter the classroom. The "profound understanding of fundamental mathematics" that Ma observed among Chinese teachers "was developed after they became teachers" (1999). The teachers she studied described several factors that helped them: "learning from colleagues, learning mathematics from students, learning mathematics by doing problems, teaching, teaching round-by-round [at different grade levels], and studying teaching materials intensively." All of these are activities that U.S. teachers can use to strengthen their own knowledge.

Starting an inquiry group.

Chinese teachers meet weekly in "teaching research groups" during which they reflect on their instructional activities and study teaching materials. Likewise, in Japan teachers participate in "lesson study groups" for at least two hours each week. In its discussion of ways to improve U.S. teachers' subject matter knowledge, the Glenn Commission recommends a similar strategy: setting up teacher "inquiry groups" that not only meet regularly during the school year, but continue through the summer as well.

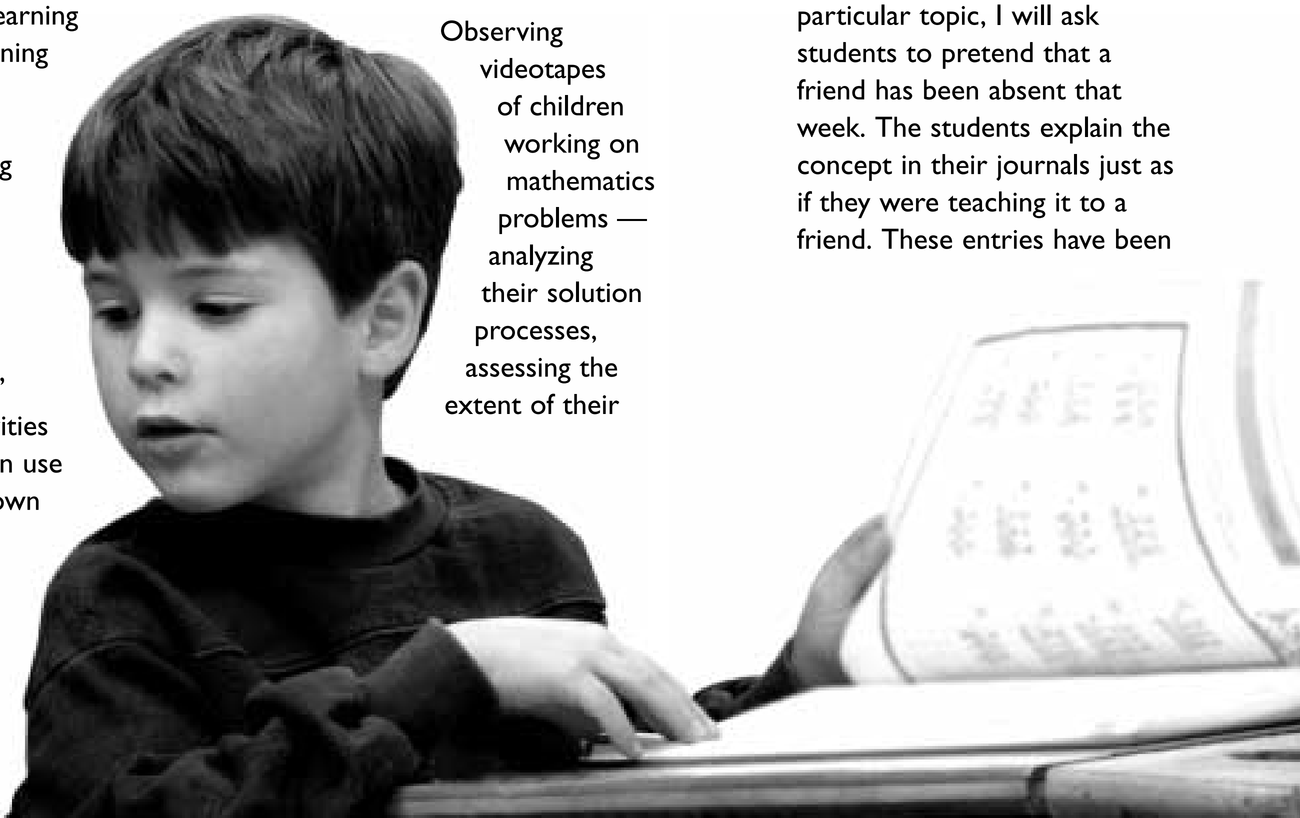
Exploring students' thinking.

Teachers can learn a lot from their own students, particularly if they structure activities and discussions in ways that uncover student strategies for thinking through a problem. One tool teachers can use in exploring and reflecting on student thinking is to keep a journal, writing down observations, recording students' problem-solving strategies, posing questions, and comparing students' activities and comments over time. Another tool is to use videotapes:

> Observing videotapes of children working on mathematics problems — analyzing their solution processes, assessing the extent of their understanding, and exploring the significance of the gaps that are exposed — can cause teachers to rethink their estimates of their students' capabilities as well as their assumptions about what "understanding mathematics" really means (Schifter, Russell, and Bastable 1999).

Teachers can also explore student learning by having students keep a mathematics journal. Zellner attributes these to providing her with a window into how her students think about mathematics, "but they have also taught me new ways of approaching problem solving…Taking time to consider what my students already know and how I can best cause them to build on that knowledge has made me look at mathematics as more of a connected whole." The journaling can take place before and/or after solving a problem:

> Students describe their plan for solving the problem and then relate the methods they tried and the results. Sometimes after several days of practice on a particular topic, I will ask students to pretend that a friend has been absent that week. The students explain the concept in their journals just as if they were teaching it to a friend. These entries have been



Getting to the Heart of the (Subject) Matter *continued*

especially helpful to me. I can quickly read through the journals and assess just how fully my students understand the concept. Any misconceptions the students may have are quickly revealed when they write. For instance, journal entries have led us to a discussion of the difference between a number, a numeral and a digit.

Enrolling in summer intensive training courses. The Glenn Commission also recommends that teachers enroll in intensive mathematics training courses during the summer months. There are a growing number of professional development opportunities focused specifically on building deep understandings of subject matter.

Start Small...But Start

The evidence is clear: We cannot significantly improve mathematics instruction without improving teachers' subject matter understanding. As Ma observes, if a teacher's own knowledge of mathematics "is limited to procedures, how could we expect his or her classroom to have a tradition of inquiry mathematics?"

It is easy to feel overwhelmed by such demands for change and point to the many external constraints that keep teachers mired in the status quo. Certainly systemic changes are needed, and teachers do not and cannot control those changes. However, it is important for each of us to start where we do have control: with our own knowledge and our own practice.

At the same time it is important not to take on too much. As educator Bruce Pirie observes about his efforts to improve his own instructional practice: "I have learned to be gentle with myself and forgiving of failure… I have not tried to transform myself over-night; if I had, I would have given up in frustration" (1997). It's a good idea to start small. Sign up for a summer workshop. Ask another math teacher to meet with you every week or two to talk about your classes. Spend an hour each week studying your teacher's guides or exploring math content on the Internet. Try having your students work in pairs to solve a problem and talk about how they went about the task.

Whatever first step you take, thing to remember is: It's the first step.

Online Mentor Program

SCIMAST mentors answered a question via the Online Mentor Program recently from a teacher who was looking for ways to increase her mathematics content knowledge through classes.

Question: I am working towards obtaining a teaching certificate in mathematics. I have a BSEE ('84) and MSCS ('89) but know that my math skills need polishing. What math courses should I take to become an effective high school math teacher.

Answer: I would think that the usual Algebra, Geometry, Calculus, etc. would be standard. However, I do know that if there is any course offered that illustrates the uses and benefits of using manipulatives, race to it! Secondary teachers usually have the content material needed, but many fall short on pedagogy. A large number of students make it to junior and senior high school without a sound basis in math. There are some great things happening in elementary math in the area of simply HOW to teach it. A GOOD class in how to teach math, even at the elementary level, would be extremely helpful for ALL teachers. Once the exposure is there, you can take the knowledge and ideas, tailor them to your needs and those of your students, and go with it. Even as an adult, I learn so much when using a variety of manipulatives to explore and explain the formulas and the whys of mathematics. If you can't find a good college level class that offers this, find an elementary teacher who excels in teaching math and visit that classroom as much as possible. It will be well worth your time, and your students will thank you for it.

Grade Level: Post-secondary

To ask a mentor a question, use this link:
http://www.sedl.org/scimast/mentoring.html.

To join the list to be notified when there is a new submission and response or to browse or search the archive, use this link:
http://www.sedl.org/scimast/archives/.

AN ACTIVITY FOR UPPER LEVEL STUDENTS

Building an Earth and Moon Model to Scale

Imagine if the Tonight Show's Jay Leno and his film crew offered you prizes to answer some simple questions on camera. Would you do it? Leno's segment always gets a laugh because the participant's answers are famous for being so embarrassingly wrong. With a similar set up, though definitely not aiming for comedy, the filmmakers for the classic video *A Private Universe* ask some Harvard students and faculty on graduation day to explain why it is warmer in the summer and why the moon has phases. In spite of significant educational expertise, they fail to successfully answer what seem like such easy questions.

The filmmakers go on to explore why simple, everyday phenomena remain so difficult to understand and explain. To investigate, they visit a nearby high school to observe students learning about the relationships between the moon, sun and earth. Despite the teacher' — and researchers' — persistent efforts, their instruction has little impact on the notions students bring to the classroom. Even after formal study, the students propose illogical and unscientific explanations, clearly demonstrating that even "simple" scientific ideas require a significant effort to understand.

How well would you do if asked these questions? Can you explain why the moon has phases? Why is the moon sometimes a bright disk of light strong enough to read by and cast shadows, while at other times it more closely resembles the disappearing smile of the Cheshire Cat from *Alice in Wonderland,* visible just long enough after the sun sets to notice before it too slips below the horizon? Where does moonlight come from, anyway?

Focusing on key concepts such as the relationships between the earth, moon, and sun are what is commonly referred to as "teaching to the big idea." Imagining the workings of the universe is more than a big idea, however, it is a huge undertaking; students can begin to understand these systems by translating them to a human scale. In this activity, they will use different sized balls to explore scale and distance using proportionality.

The fluctuations of the moon have intrigued the human race for as long as we have records. Over 30,000 years ago, some ancient ancestor carved notches on a bone that tracked the waxing (growing) and waning (fading) of the moon's bright light. Historians argue that the moon's changing aspect helped inspire humans to develop a construct of time itself. The moon still continues to be honored as the second day of the week (Monday) and the source for the divisions of the year (month).

Writers are famous for incorporating the moon into their works. Who could forget the Shakespeare's *Romeo and Juliet* balcony scene?

ROMEO:
Lady, by yonder blessed moon
I swear
That tips with silver all these
fruit-tree tops —

JULIET:
O, swear not by the moon,
the inconstant moon,
That monthly changes in her
circled orb,
Lest that thy love prove
likewise variable.

Galileo trained his first telescope on the moon's changing face in the early 1600's and observed rugged features emerging from the darkness as the lunar terminator (edge of night) moved across the disk. It reminded him of the way that mountains on earth catch the first rays of the dawn, suggesting that the moon and the earth were very similar celestial bodies. He argued that the phases of the moon were not unique in the solar system but should appear for any object that orbits the sun. His telescope confirmed that Venus and, to a lesser extent, Mars also had phases, providing a critical confirmation of Copernicus' sun-centered solar system. When he turned his telescope on Jupiter and saw four bright moons orbiting the giant planet, the Earth suddenly lost its place as the center of the universe.

What do we know about our closest neighbor in space? How big is the moon? How far is it from the earth? Why does it sometimes cover the sun, creating a solar eclipse? Few people can explain why the moon appears to change over the course of any month, actually disappearing from view before reappearing at the other side of the sun.

This activity uses simple models to investigate the moon/earth/sun system, creating a memorable understanding of their distance and size relationship. Using this new perspective, students can explore the geometric relationships that cause the observed phenomenon we call the phases of the moon, the scale of the solar system, and beyond.

Building an Earth and Moon Model to Scale *continued*

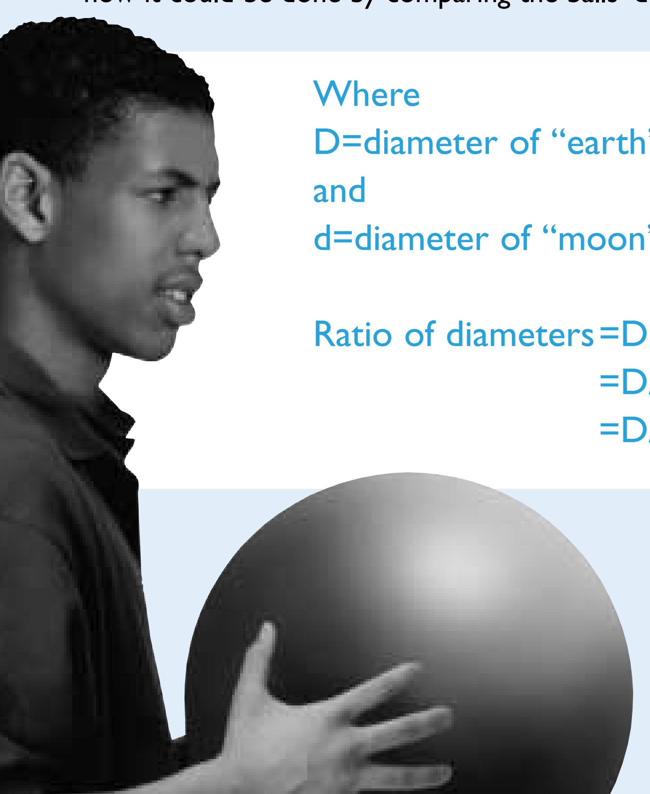
Activity Time Frame: One to two class periods

Understanding both the relative sizes and the scale distance between the earth and moon provides a concrete foundation for more complex conceptual understandings of the earth/moon system. In this activity, students explore their understandings of the size and distance relationships using an assortment of different dimensioned balls. Materials to have on hand include metric rulers and a collection of assorted sized balls — from marbles to beach balls.

Group students into teams of two to three. There should be at least two balls for each team — the more, the better. Discuss "scale" in terms of distance and relationship. It may be helpful to have a scale model or map to explain the concept. Teams choose two balls from the collection that represent their best estimate of the scaled relationship between the size of the earth and moon. Have student teams explain the logic (or lack of) behind their choices.

Student choices (and explanations) will be highly variable. Obviously, they cannot all be right. To reduce variability, have students develop a standard ratio to help them compare the sizes of the earth and moon.

Ask the class if anyone knows how to determine a ratio to describe the relationship between the sizes of their "earth" and "moon". Students might propose several unique ways, including volume or circumference. Discuss how it could be done by comparing the balls' diameters.



Where
D=diameter of "earth" ball
and
d=diameter of "moon" ball

Ratio of diameters $= D:d$
$= D/d : d/d$
$= D/d : 1$

Each team should use a ruler to measure the diameters of their two balls and complete the equation, arriving at some ratio like 5:1 or 4:1 or 3:1. Record each team's ratio. Ask if their information is enough to resolve the differences between answers.

Provide them with the following actual values and have them complete the ratio exercise once more to find the approximate actual ratio. If you divide 12,800 by 3,200, you get a 4:1 ratio.

Earth	D	$\approx$	12,800	kilometers
Moon	d	$\approx$	3,200	kilometers

After students calculate the approximate ratio, have them search for two balls that have a 4:1 ratio. State that the "earth" and "moon" they now hold represent the earth/moon model in correct size scale relationship. Students holding bigger "earths" should have a "moon" that is proportionally correct.

Now that sizes are relatively accurate, what about distance? To physically model distance, each student should hold one ball and stand as far apart as the team thinks the earth and moon are. Remind them to consider the scale of their model orbs. You may want to move outside to do this, though most classrooms are big enough. Again, expect a great deal of variation in student representations.

Ask students how they might use what they know about scale to estimate an approximate distance relationship for the model balls. If students cannot solve the problem, you might assist them by suggesting that the diameter of the earth is about 12,800 km and the distance to the moon from the earth is about 384,000 kilometers. Dividing 384,000 km. By 12,800 km. yields 30, suggesting the moon's distance is about 30 earth diameters away. If the student model selected had an earth diameter of 10 cm., then the scale distance would be 30 x 10 cm. = 300 cm., or 3 meters!

We could build an equation that relates these numbers as follows:

$$\frac{\text{Distance from earth to moon}}{\text{Diameter of earth}} = \frac{\text{X (scale distance)}}{\text{Diameter of earth model}}$$

$$\frac{\text{384,000 km.}}{\text{12,800 km.}} = \frac{\text{X (scale distance)}}{\text{Y cm. (from student example)}}$$

$$30\ (\text{Y cm.}) = \text{Scale distance}$$

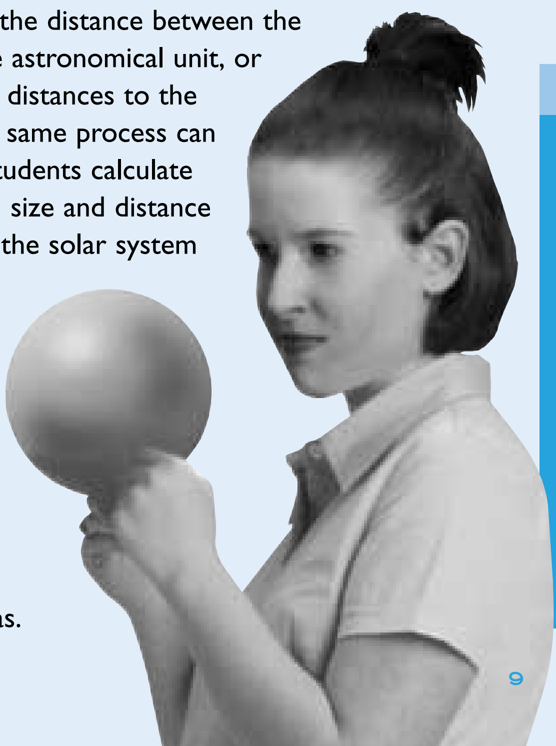
Have each team calculate how far apart their two balls should be and then stand the correct distance apart. Remind the students that the distance they stand apart is determined by the size of the ball they choose to represent the earth in their model. The larger the ball, the further apart they must stand.

The two scale relationships calculated here are much simpler to remember than the actual numbers: the earth is four times larger than the moon, and the distance between the two is about 30 earth diameters.

Scale models help us to imagine relationships that we cannot see or even imagine. There are many places students can go from here. Three important ideas your students may wish to explore once they have physical scale models with which to work include the orbit of the moon around the earth, eclipses and phases of the moon, and the scale distance to the sun and other planets.

The orbit of the moon around the earth can be easily explored by moving the model moon around the model earth. A string can be used to keep the two about 30 diameters apart. If this is done outside on a sunny day, the student holding the earth ball will observe the phases of the moon on the model. Since the moon orbits in a plane that takes it very close to the apparent location of the sun in the sky, this is most easily done in the early morning while the sun is still low. Another way to model the phases is clearly described in the GEMS activity *Modeling Moon Phases and Eclipses: Observing Moon Phases* (see resources).

Additionally, students interested in calculating the scale distance to the Sun can do it the same way they did the distance between the earth and moon above, using a value of about 150 x 10^6 km. for the distance. Astronomers call the distance between the earth and sun one astronomical unit, or AU for short. For distances to the other planets, the same process can be used. Having students calculate the correct scaled size and distance for the planets of the solar system and holding these balls the proper distance apart in a large field is a powerful way to help students build and relate lasting concrete understandings of very complex ideas.



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Web Sites on Modeling Moon Phases

http://seds.lpl.arizona.edu/nineplanets/nineplanets/nineplanets.html

http://www.astro.wisc.edu/~dolan/java/MoonPhase.html

http://www.ac.wwu.edu/~stephan/phases.html

http://csep10.phys.utk.edu/astr161/lect/time/moonorbit.html

http://www.netaxs.com/~mhmyers/moon.tn.html

National Standards

Principles and Standards for School Mathematics

The very first standard set forth and covering all grades (pre-K through 12th) in the Standards of the National Council of Teachers of Mathematics concerns Number and Operations. The following are excerpted from the NCTM's *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Instructional programs from prekindergarten through grade 12 should enable all students to —

- **understand numbers**, ways of representing numbers, relationships among numbers, and number systems;
- **understand meanings** of operations and how they relate to one another;
- **compute fluently** and make reasonable estimate.

Central to this standard is the development of number sense — the ability to decompose numbers naturally, use particular numbers like 100 or $^1/_2$ as referents, use the relationships among arithmetic operations to solve problems, understand the base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers (Sowder 1992).

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

The understanding of numbers develops in prekindergarten through grade 2 as children count and learn to recognize "how many" in sets of objects. A key idea is that a number can be decomposed and thought about in many ways. For instance, 24 is 2 tens and 4 ones and also 2 sets of twelve. Making a transition from viewing "ten" as simply the accumulation of 10 ones to seeing it both as 10 ones and as 1 ten is an important first step for students toward understanding the structure of the base-ten number system (Cobb and Wheatley 1988).

Beyond understanding whole numbers, young children can be encouraged to understand and represent commonly used fractions in context, such as 1/2 of a cookie or 1/8 of a pizza, and to see fractions as part of a unit whole or of a collection.

Representing numbers with various physical materials should be a major part of mathematics instruction in the elementary school grades. By the middle grades, students should understand that numbers can be represented in various ways, so that they see that $^1/_4$, 25%, and 0.25 are all different names for the same number.

As students gain understanding of numbers and how to represent them, they have a foundation for understanding relationships among numbers. As their number sense develops, students should be able to reason about numbers by, for instance, explaining that $^1/_2$ + $^3/_8$ must be less than 1 because each added is less than or equal to $^1/_2$. In grades 6–8, it is important for students to be able to move flexibly among equivalent fractions, decimals, and percents and to order and compare rational numbers using a range of strategies.

Understand meanings of operations and how they relate to one another

During the primary grades, students should encounter a variety of meanings for addition and subtraction of whole numbers. Researchers and teachers have learned about how children understand operations through their approaches to simple arithmetic problems like this:

Bob got 2 cookies. Now he has 5 cookies. How many cookies did Bob have in the beginning?

To solve this problem, young children might use addition and count on from 2, keeping track with their fingers, to get to 5. They might also recognize this problem as a subtraction situation and use the fact that 5 – 2 = 3. Exploring thinking strategies like these or realizing that 7 + 8 is the same as 7 + 7 + 1 will help students see the meaning of the operations.

In grades 3–5, helping students develop meaning for whole-number multiplication and division should become a central focus. By creating and working with representations (such as diagrams or concrete objects) of multiplication and division situations, students can gain a sense of the relationships among the operations.

In grades 6–8, operations with rational numbers should be emphasized. Students' intuitions about operations should be adapted as they work with an expanded system of numbers (Graeber and Campbell 1993).

In grades 9–12, as students learn how to combine vectors and matrices arithmetically, they will experience other kinds of systems involving numbers in which new properties and patterns emerge.

Compute fluently and make reasonable estimates

Experience suggests that in classes focused on the development and discussion of strategies, various "standard" algorithms either arise naturally or can be introduced by the teacher as appropriate. The point is that students must become fluent in arithmetic computation — they must have efficient and accurate methods that are supported by an understanding of numbers and operations. "Standard" algorithms for arithmetic computation are one means of achieving this fluency.

Part of being able to compute fluently means making smart choices about which tools to use and when. Students should have experiences that help them learn to choose among mental computation, paper-and-pencil strategies, estimation, and calculator use. The particular context, the question, and the numbers involved all play roles in those choices. Do the numbers allow a mental strategy? Does the context call for an estimate? Does the problem require repeated and tedious computations? Students should evaluate problem situations to determine whether an estimate or an exact answer is needed, using their number sense to advantage, and be able to give a rationale for their decision.

"Know How, and Also Know Why": A Look at Liping Ma's Research on Mathematics Teaching

Comparing mathematics teaching and learning in China and the United States, researcher Liping Ma was struck by a paradox: Although American elementary mathematics teachers receive between 16 and 18 years of formal schooling, U.S. students are consistently outperformed by students from many other countries, including China, where teachers have had typically only 11 or 12 years of formal schooling. To investigate, Ma interviewed teachers in the U.S. and China. She engaged teachers with four mathematics problems typically taught in elementary school, asking them to solve the problem and describe the mathematical basis for the solution. She reports these results in *Knowing and Teaching Elementary Mathematics*.

Subtraction with Regrouping

Ma first asked teachers how they would work with second graders who were trying to solve a problem such as 52 – 25, and what students would need to know in order to be able to solve the problem. She found that 77 percent of U.S. and 14 percent of Chinese teachers "displayed only procedural knowledge of the topic." For these teachers, "their understanding was limited to surface aspects of the algorithm — the taking and changing steps. This limitation in their knowledge confined their expectations of student learning as well as their capacity to promote conceptual learning in the classroom" (Ma 1999, 27).

Multidigit Multiplication

Next, Ma presented a scenario in which sixth-grade students, when trying to multiply large numbers such as 123 x 645, were forgetting to move the partial products over on each line. She asked teachers what they would do to help students who were making this mistake. With this problem as well, all the teachers knew the correct procedure. However, nearly two-thirds of U.S. teachers were unable to provide a conceptual explanation. Ma found that

> …teachers had different views of the problem: Some considered it a problem of knowing the procedure; others thought it a problem of conceptual understanding. The teachers' perspectives on the problem paralleled their subject matter knowledge of the topic (54).

Division by Fractions

The third problem involved division by fractions. Ma asked teachers to solve the problem, $1\,^3/_4 \div {}^1/_2$, and to create a story to illustrate the problem. Results here were particularly striking: Less than half

"Know How, and Also Know Why" *continued*

of the U.S. teachers were able to correctly calculate a complete answer to this problem, and none showed an understanding of the rationale underlying their calculations. Only one U.S. teacher was able to generate an appropriate story problem. In contrast, all of the Chinese teachers' calculations were correct, and 90 percent were able to generate a conceptually correct story problem.

Moreover, the U.S. teachers who did solve the problem used only one approach: "invert and multiply — the standard algorithm" (60-61). Chinese teachers, however, proposed at least three additional problem-solving approaches and went on to talk about the need for students to be able to multiply with fractions in order to understand division with fractions.

The Relationship between Perimeter and Area

The final problem Ma posed to teachers was a situation in which a student tells her teacher that, on her own, she has discovered a theory: As the perimeter of a closed figure such as a rectangle increases, the area also increases. Ma asked teachers how they would respond to the student.

She found that teachers' initial reactions were similar: Most talked about the importance of encouraging the student's independent exploration of mathematics concepts. All the teachers knew the meaning of "perimeter" and "area," and most knew how to calculate them. A similar proportion of U.S. and Chinese teachers accepted the students' theory.

From this beginning, however, the teachers' paths diverged. They explored different strategies, reached different results, and responded to the student differently (85).

U.S. teachers often wanted to "look up" the answer in a textbook; Chinese teachers did not. U.S. teachers tended to rely on "the idea that a mathematical claim should be proved by a large number of examples" (90). Most of the Chinese teachers explored the problem mathematically on their own, while only one U.S. teacher investigated the problem and reached a solution.

In the bulk of the text, Ma develops a clear description of the differences between the U.S. teachers and their practices and the Chinese teachers and their practices. She characterizes the Chinese teachers as more understanding of the mathematics they are teaching, more capable of teaching it in multiple ways, and more understanding of the errors students might make in learning the mathematics and why they might make them.

Some important points to consider include:

- U.S. teachers describe mathematics in elementary school as simple, while Chinese teachers think of elementary mathematics as "fundamental." Unpacking the complex concept of fundamental mathematics, Ma describes school mathematics as foundational, primary, and elementary.
  Foundational — provides a foundation on which advanced study is built.
  Primary — contains the rudiments of many important concepts developed later in more advanced branches.
  Elementary — provides the beginning, the introduction, as opposed to the simple version of the ideas of mathematics.

- Ma argues that the highest quality teachers should have a Profound Understanding of Fundamental Mathematics (PUFM) and describes such understandings as deep, broad, and thorough.
  - Understanding a topic with **depth** enables teachers to connect it to the more conceptually powerful ideas of the subject.
  - Understanding a topic with **breadth** enables teachers to connect it with other topics of similar or less conceptual power.
  - Depth and breadth depend on **thoroughness** — the capacity to 'pass through' all parts of the field — to weave them together.

- Experienced teachers in China outperformed new teachers. Some of the Chinese teachers, the most experienced, had what she would call PUFM. None of the U.S. teachers exhibited this level of mathematical knowledge. In the U.S., experienced teachers performed no differently than new teachers. This leads Ma to conclude that the continued study of mathematics throughout a teacher's career is critical to developing fundamental understandings.

Profound Understanding of Fundamental Mathematics

PUFM

Ma argues that "the quality of teacher subject matter knowledge directly affects student learning"; therefore, improving mathematics education for students would result from "improving the quality of their teachers' knowledge of school mathematics" (144). Ma suggests five strategies for accomplishing this objective.

- **Address teacher knowledge and student learning at the same time**. "Because they are interdependent processes, we cannot expect to improve teachers' mathematical knowledge first, and, in so doing, automatically improve students' mathematics education" (147).

- **Enhance the interaction between teachers' study of school mathematics and how to teach it.** Two assumptions hinder U.S. teachers' continuing study of mathematics: 1) elementary mathematics is "basic," superficial, and commonly understood; and 2) teachers do not need further study of the subject they teach. Given current conditions, it is unlikely that U.S. teachers will be able to gain the needed skills. "What U.S. teachers are expected to accomplish, then, is impossible. It is clear that they do not have enough time and appropriate support to think through thoroughly what they are to teach. And without a clear idea of what to teach, how can one determine how to teach it thoughtfully?" (149).

- **Refocus teacher preparation**. "What we should do is to rebuild a substantial school mathematics with a more comprehensive understanding of the relationship between fundamental mathematics and new advanced branches of the discipline" (149).

- **Understand the role that curricular materials, including textbooks, might play in reform**. "Teachers need not have an antagonistic relationship with textbooks. My data illustrate how teachers can both use and go beyond the textbook" (150).

- **Understand the key to reform: whatever the form of classroom interactions might be, they must focus on substantive mathematics**. Ma challenges reformers to look beyond the rhetoric of reform and the superficial manifestations in the classroom, arguing that Chinese classrooms "look traditional" while many U.S. classrooms have the appearance of being driven by the values of reform. "If you look carefully at the kind of mathematics that the Chinese students are doing and the kind of thinking they have been encouraged to engage in, and the way in which the teacher's interactions with them foster that kind of mental and mathematics process, the two kinds of classrooms are actually much more similar than they appear" (153). This point is critical, since "the real mathematics thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics" (153).

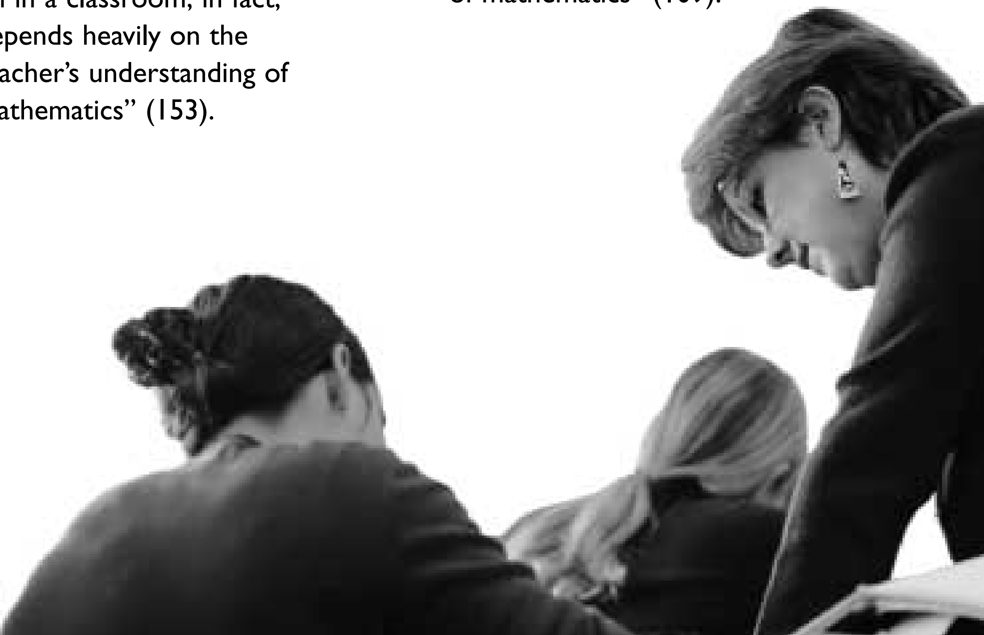
Not Just What, but Why

From this research, Ma concludes that:

> Limited subject matter restricts a teacher's capacity to promote conceptual learning among students. Even a strong belief of "teaching mathematics for understanding" cannot remedy or supplement a teacher's disadvantage in subject matter knowledge (36).

Her findings from other studies confirm that "what teachers expected students to know…was related to their own knowledge. The teachers who expected students merely to learn the procedure" — as did most of the U.S. teachers in the study — "tended to have a procedural understanding" (3).

In contrast, most of the Chinese teachers tended to emphasize conceptual understanding, to explore problems in the ways mathematicians themselves do, and to focus on the connections among mathematical ideas and the basic knowledge on which more complex understandings are built. In addition to considering what (i.e., algorithms and answers), Chinese teachers emphasized why. Ma notes, "The predilection to ask 'Why does it make sense?' is the first stepping stone to conceptual understanding of mathematics" (109).



Access Centers

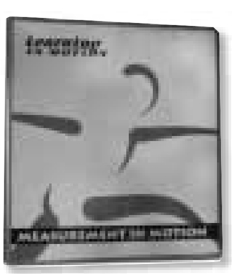
NEW Resources at Local Access Centers You Can Borrow for Free!

SCIMAST has partnered with education organizations to provide teachers with 40 Access Centers throughout the Southwest. Access Centers act as libraries for education resources, offer training on materials that support mathematics and science instruction, and network sessions for teachers to share what they learn. Lending policies and hours vary depending on the center. For more information or to search for a resource, visit the access centers online at http://www.sedl.org/scimast/accesscenters/.

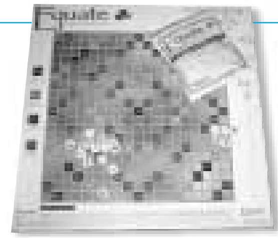
This Spring, Access Center representatives met to choose new materials relating to Mathematical Design, Constancy and Change, Data Collection and Analysis, and Systems. Here are some favorites (note: resources vary at each Access Center):

Measurement in Motion

Learning in Motion



This kit contains software and a teacher's guide with activities that enable students to mathematically and scientifically analyze motions that are common in their lives. This kit lets students connect concrete situations in movies directly to visual data representations in graphs and charts, and to symbolic representations in data tables and formulas.



Equate

Conceptual Math Media

This math-oriented board game requires computing and strategic thinking and can be played at several skill levels for two to four players or teams. Using hands consisting of nine tiles, representing numbers and operations, players form horizontal and vertical equations by placing tiles on the board. Each successive play must connect with a previous play. The game is designed to improve thinking skills, develop number sense, and build a foundation for success in algebra.

Transforming Functions to Fit Data

R. J. Carlson and M. J. Winter

This activity book with a CD-ROM contains activities and explorations in which students use data collector probes and graphing calculators to gather data and transform equations to fit the data. In these experiments, students, from prealgebra to precalculus, explore functions and investigate the interrelationships among graphs, tables, algebraic expressions, and physical conditions.

Force and Motion: Hands On Science Series

S. Souza and K. Kwitter

The lab and activities in this book open the door of discovery to students, exploring the fundamental properties of objects at rest and in motion. The hands-on materials provide a fun and easy way to investigate vectors, Newton's laws of motion, friction, gravitation, kinetic and potential energy, and other key concepts in physics.

Ecology and Evolution: Islands of Change

NSTA

Using the Galapagos Islands as the basis for its hands-on, inquiry oriented activities for students in grades 5 to 8, this book attempts to demonstrate how ecology and evolution are inherently linked. The concepts covered in the activities include geologic time scale, island characteristics that impact ecological relationships, and the factors that lead to evolution.

Assessing Toxic Risk — Teacher and Student Guides

N. M. Trautmann

This guide helps students conduct environmental research while they learn the basics of toxicology and the use of bioassays as a measuring tool. The activities steer the class through the steps of learning research techniques. Thanks to the cross-disciplinary approach of these books, students learn firsthand the links between biology, chemistry, environmental science, and human health.

Inquire Within: Implementing Inquiry Based Science Standards

D. Llewellyn

Written for conscientious educators seeking to cultivate science learners' sense of discovery and critical thinking skills, *Inquire Within* offers enlightening guidelines and information.

Resources and Opportunities

1. The Number Devil

Hans Magnus Enzensberger

A delightful book that makes mathematics fun, The Number Devil centers around a self-described math-phobe who begins having surreal dreams in which he encounters a number devil. The number devil teaches him, for starters, the crucial importance of zero and how to make all numbers out of ones. More difficult concepts are deliberately repeated in several dreams; occasional additional problems are provided for greater challenges.

2. Teaching Children Mathematics

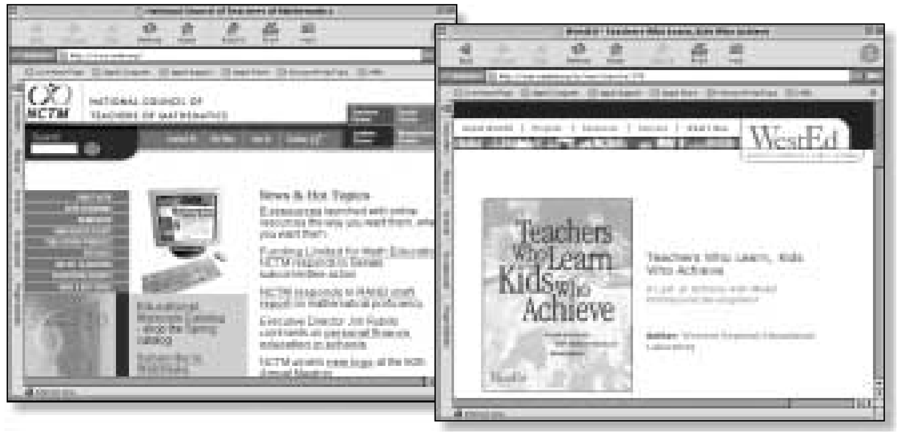
NCTM

An NCTM journal with activities, lesson ideas, teaching strategies, and in-depth articles. Past article examples include "Learning Geometry in a Dynamic Computer Environment" and "Calculators as Learning Tools for Young Children's Explorations of Number."
http://www.nctm.org/

3. A Collection of Math Lessons

Marilyn Burns & Bonnie Tank

This book contains 14 lessons exploring estimation, word problems, multiplication, fractions, patterns, statistics, probability, geometry, and measurement. This book presents a lively, readable classroom vignette that describes a unique and inspiring approach to teaching problem-solving lessons.



4. Atlas of Science Literacy

American Association for the Advancement of Science, NSTA Press

What should students learn? When? In what order? How does each strand of knowledge connect to other vital threads? This book provides a road map to help students learn science systematically. The book traces the prerequisites for learning in each grade, making the connections to support science content, and showing the way to the next steps to learning for your students. Fifty linked maps show exactly how students K–12 can expand their understanding and skills toward specific science-literacy goals.
http://store.nsta.org/

5. Teacher Change: Improving K–12

ENC

Available from the ENC as a CD-ROM or online, this resource collection is designed to help educators and professional development providers facilitate discussion and reflection on improving K-12 mathematics and science. It includes professional development activities, selected materials from the collection, and stories from math and science teachers about the process of change.
http://www.enc.org/professional/learn/change/

6. Teachers Who Learn, Kids Who Achieve: A Look at Schools with Model Professional Development

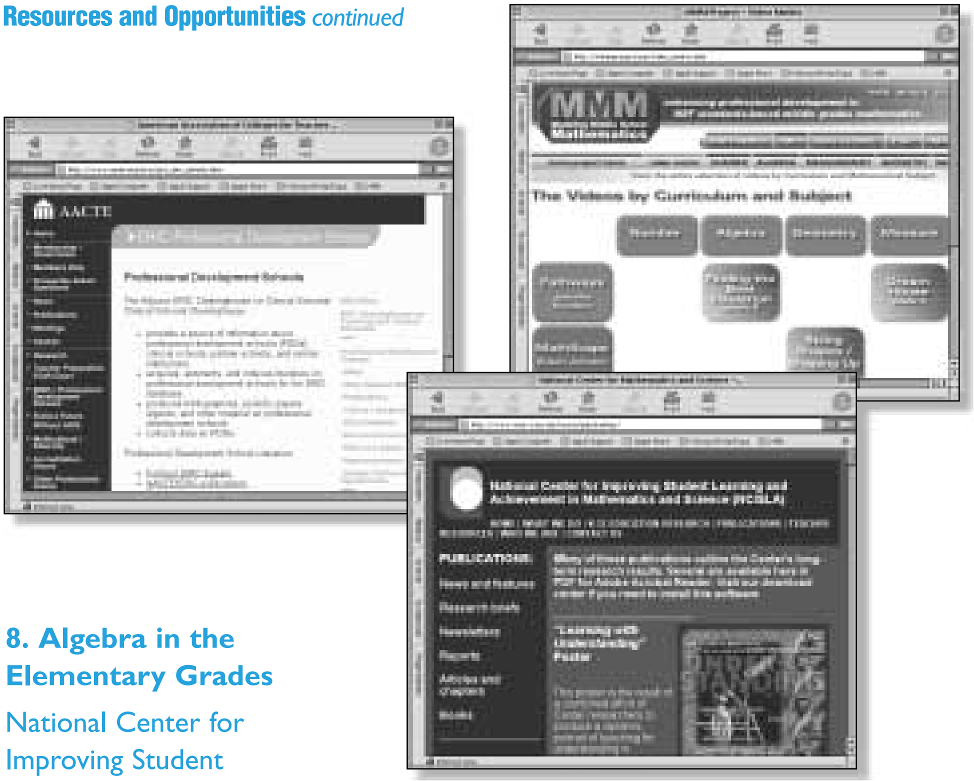
Western Regional Educational Laboratory

A research study of eight schools that won a federal award for Model Professional Development has been distilled into this brief and compelling story of successful school reform. Teacher voices and vignettes give life to the guiding principles that researchers identified across disparate sites. Annotated lists of resources provide concrete help in putting these principles into practice, and profiles of each school's journey demonstrate that extraordinary results can be achieved from even modest beginnings. Available as a book or online as a PDF or HTML document at http://web.wested.org/cs/wew/view/rs/179

7. Modeling Middle School Mathematics

This professional development program has lessons available covering statistics, algebra, geometry, and more. Organized by curriculum and subject, you can view online video lessons, transcripts, teacher pages, and student work from real teachers and their classrooms.
http://mmmproject.org/video_matrix.htm

Resources and Opportunities *continued*



8. Algebra in the Elementary Grades

National Center for Improving Student Learning and Achievement in Mathematics and Science

In the newsletter *In Brief* and available online as a PDF, this article is for policymakers, educators and researchers seeking to improve student learning and achievement; it reports on Center research and its implications for mathematics and science reform. Teachers and researchers have long recognized that the transition from learning arithmetic to learning algebra is one of the major hurdles students face in learning mathematics. Researchers are finding, however, that elementary students can learn to think about arithmetic in ways that both enhance their early learning of arithmetic and provide a foundation for learning algebra. This newsletter highlights learning gains of 240 elementary students in a long-term study and their ability to reason about arithmetic in ways that build their capacity for algebraic reasoning.

http://www.wcer.wisc.edu/ncisla/publications/briefs/fall2000.pdf

9. American Association of Colleges for Teacher Education Web site

This Web site hosts the ERIC Clearinghouse that includes research digests, answers to frequently asked questions, and a sample partnership agreement for schools and universities.

http://www.aacte.org/eric/pro_dev_schools.htm

10. By Your Own Design

The ENC and the National Staff Development Council created this CD-ROM and Web site as a professional learning guide to assist teachers in building a foundation for professional learning (such as taking stock in time and funding issues and building a learning community), selecting learning strategies (such as increasing subject knowledge, improving curriculum, and using technology), and measuring results.

http://enc.org/professional/guide/

11. Thinking Mathematics

Thinking Mathematics is an American Federation of Teachers (AFT) research-based, teacher-developed approach to teaching mathematics. It focuses on sense-making ways to think about mathematics for teachers and students, particularly in grades K-6. Summer Institutes provide courses for teachers sponsored by school districts. AFT staff visit each new site during the start-up year to provide support in the form of coaching, demonstration lessons, assistance with local professional development, contact with administration or local colleges, or whatever the local need. The basic course is contained in two volumes: 1) Foundations showing how principles can be applied to counting, addition, and subtraction; 2) Extensions, which applies the principles to multiplication, division, and beginning proportional reasoning. For more information, visit the Web site:

http://www.aft.org/thnkmath/thmth1.htm

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